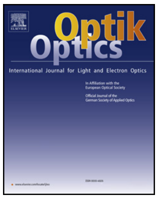
Optik - International Journal for Light and Electron Optics 206 (2020) 164344

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo



Original research article

Chirped super-Gaussian and super-sech pulse parameter dynamics with DWDM topology by variational principle

Amour M. Ayela[a], Gaston Edah[b,c], Anjan Biswas[d,e,f,g], Mehmet Ekici[h,*], Abdullah Kamis Alzahrani[e], Milivoj R. Belic[i]

[a] Institute of Mathematical and Physical Science, University of Abomey-Calavi, Bénin
[b] Faculty of Science and Technology, University of Abomey-Calavi, Bénin
[c] International Chair of Mathematical Physics and Application, Bénin
[d] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762-4900, USA
[e] Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[f] Department of Applied Mathematics, National Research Nuclear University, 31 Kashirskoe Shosse, Moscow 115409, Russian Federation
[g] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[h] Department of Mathematics, Faculty of Science and Arts, Yozgat Bozok University, 66100 Yozgat, Turkey
[i] Science Program, Texas A&M University at Qatar, PO Box 23874, Doha, Qatar

ARTICLE INFO

OCIS:
060.2310
060.4510
060.5530
190.3270
190.4370*Keywords:*
Log-law nonlinearity
Multiplexing
Lagrangian variational method

ABSTRACT

This paper studies multiplexing phenomenon with logarithmic nonlinearities during propagation of ultra-short optical pulses in an optical fiber with several different channels of refractive index. This study is based on the resolution by the Lagrangian variational method of the nonlinear Schrödinger's equation with log-law. The dynamical system of parameter evolution with super-Gaussian and super-sech functions is presented.

1. Introduction

Optical Gaussons is one of the lesser known type of solitons that are frequented in optoelectronics for the past few decades [1–20]. These type of solitons are typically visible in the governing nonlinear Schrödinger's equation appears with logarithmic law nonlinearity [1–5]. One advantage of Gaussons over conventional solitons is that these do not produce soliton radiation. These phenomena are used in the application of the materials. The most common application is the use of base materials for the manufacture of waveguides that can be used in the field of guided optics [5]. This part of the nonlinear optics is based essentially on the constitution and operation of optical information transmission and telecommunication devices. Speaking of the constitution, the waveguides are made using a stack multilayer of different refractive index on planar substrate [5]. The constitution of the waveguides makes it possible to predict certain physical phenomena and, in this case, double refraction phenomena are expected which confers the birefringent character on the two-layer waveguides. Thus for each nonlinear effect we associate a sum of directional responses given the variety of the refractive index. This makes it possible to bridge the nonlinearities of optics and mechanics. For this purpose, the nonlinear response, which is shown here as a sum of directional responses, is also seen as a sum of the wave function of each of the

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[*] Corresponding author.
E-mail address: mehmet.ekici@bozok.edu.tr (M. Ekici).

https://doi.org/10.1016/j.ijleo.2020.164344
Received 23 December 2019; Accepted 31 January 2020

constituent particles of the mechanical study system. Since the log-law nonlinearity was already introduced into the 1976 Schrödinger equation by Birula and Mycielski to study the nonlinear responses [6], it would be good to think of a similar law. Models that describe logarithmic nonlinearities in nonlinear optics exist in several forms. Thus we will give some of the equations of study of logarithmic nonlinearities which are as follows.

2. Governing model

$$\mathrm{i}q_t + \mathrm{a}q_{\mathrm{xx}} + \mathrm{b}q\log|q|^2 = 0; \tag{1}$$

where a and b are constants. This equation modeled by Guerrero et al. [7] has already been treated by Khalique and Biswas who have proposed a Gaussian soliton solution to this Schrödinger equation [8]. The same equation was treated by Girgis et al. using a Gaussian test function [9]:

$$i\psi_z = -\psi_{\mathrm{xx}} + V\psi + g\psi\log|\psi|^2; \tag{2}$$

where $V = V(z, x)$ and $g = g(z)$ are respectively linear and non-linear coefficients of the Schrödinger equation.

This equation has been solved by using a trial function [10]:

$$\mathrm{i}q_z^{(l)} + a_l q_{\mathrm{tt}}^{(l)} + i\alpha_l q_t^{(l)} + \left\{ b_l \log\,|q^{(l)}|^2 + \sum_{n\neq l}^{N} \lambda_{\mathrm{ln}} \log\,|q^{(n)}|^2 \right\} q^{(l)} = 0. \tag{3}$$

Eq. (1), treated by Biswas [14], will be in our work solved by the Lagrangian variational method in order to find the Lagrangian that describes this physical system and the variational equations that can describe the evolution of the amplitude, the inverse of width, the center position, the chirp, the frequency and the phase.

3. Preliminaries

Let us rewrite Eq. (3)

$$\mathrm{i}q_z^{(l)} + a_l q_{\mathrm{tt}}^{(l)} + i\alpha_l q_t^{(l)} + \left\{ b_l \log\,|q^{(l)}|^2 + \sum_{n\neq l}^{N} \lambda_{\mathrm{ln}} \log\,|q^{(n)}|^2 \right\} q^{(l)} = 0. \tag{4}$$

The starting point for the Lagrangian variational method consists of the calculation of the Lagrangian density. The Lagrangian density L_0 of this system may be cast into the form:

$$\begin{aligned} L_0 &= \frac{i}{4}(q_z^{(l)} q^{(l)*} - q_z^{(l)*} q^{(l)}) + \frac{i\alpha}{4}(q_t^{(l)} q^{(l)*} - q_t^{(l)*} q^{(l)}) - \frac{a}{2}|q_t|^2 \\ &\quad + \frac{b_l}{2}[|q^{(l)}|^2 \log|q^{(l)}|^2 - 1] + \sum_{n\neq l}^{N} \lambda_{\mathrm{ln}}\,|q^{(l)}|^2 \log|q^{(n)}|^2. \end{aligned} \tag{5}$$

Therefore, the average Lagrangian L is defined as:

$$L = \int_{-\infty}^{+\infty} L_0\,\mathrm{dt}. \tag{6}$$

Now, supposing that the solution of this system is given by a chirped pulse as

$$q^l(z, t) = A_l f(B_l(t - t_l))\exp\left[i\left(\frac{C_l}{2}(t - t_l)^2 + k_l(t - t_l) + \theta_l \right) \right], \tag{7}$$

where f stands for the shape of the pulse that is going to be the super-sech and the super-Gaussian type in this study. Also, the parameters $A_l(z)$, $t_l(z)$, $B_l(z)$, $C_l(z)$, $k_l(z)$ and $\theta_l(z)$ represent the amplitude of the soliton, the center of the pulse, the inverse width of the pulse, chirp, frequency and the phase of the pulse respectively.

A set of evolution equations for the pulse parameters will be derived employing the Lagrangian variational principle. Let's note that, this approach is only approximate and does not account for characteristics such as energy loss due to continuum radiation, damping of the amplitude oscillations and changing of the pulse shape. For convenience, the following integral is defined:

$$I_{i,j,k}^{(l)} = \int_{-\infty}^{+\infty} \tau^i f^j(\tau) \left(\frac{\mathrm{df}}{d\tau} \right)^k d\tau, \tag{8}$$

where i, j and k are non-negative integers. Next utilizing (7), the Lagrangian (6) becomes

$$\begin{aligned} L \;=\; & -\frac{1}{4}A_l^2 C_l' \frac{I_{2,b,0}}{B_l^3} + \frac{1}{2}(A_l^2 k_l t_l' - A_l^2 \theta_l')\frac{I_{0,2,0}}{B_l} - \frac{\alpha}{2}A_l^4 k_l \frac{I_{0,2,0}}{B_l} \\ & - \frac{a_l}{2}A_l^2 C_l^2 \frac{I_{2,2,0}}{B_l^3} - \frac{a_l}{2}A_l^2 k_l \frac{I_{0,2,0}}{B_l} + 2A_l^2 \frac{I_\lambda}{B_l} + (2A_l^2 \log(A_l) - A_l^2)I_{0,2,0} \\ & + \frac{1}{2}\sum_{n\neq l}^{N} \lambda_{\ln} A_n^2 J_{\ln} \frac{a_l}{2} A_l^2 B_l^3 I_{0,0,b}; \end{aligned} \tag{9}$$

with

$$J_{\ln} = \int_{-\infty}^{+\infty} \prod_{j=l,n} f^2[B_j(z)(t - t_j(z))]\mathrm{dt}. \tag{10}$$

Euler–Lagrange equation of the shape is given by:

$$\frac{\partial L}{\partial P(z)} - \frac{d}{\mathrm{d\,z}}\frac{\partial L}{\partial \dot{P}(z)} = 0, \tag{11}$$

with $P = \{A_l,\, t_l,\, B_l,\, C_l,\, k_l,\, \theta_l\}$; $\dot{P} = \frac{d}{\mathrm{dz}}P(z)$, and while implementing the relation (11) one gets the following dynamical system:

- for the soliton amplitude

$$A_l' = \mathrm{aA}_l C_l, \tag{12}$$

- for the center position

$$t_l' = -(a + \alpha A_l^2), \tag{13}$$

- for the width

$$B_l' = 2\mathrm{aB}_l C_l, \tag{14}$$

- for the chirp

$$\begin{aligned} C_l' \;=\; & -2\alpha A_l^2 B_l^2 k_l \frac{I_{0,2,0}}{I_{2,2,0}} - 4\mathrm{aB}_l^6 \frac{I_{0,0,2}}{I_{2,2,0}} + 2\mathrm{aC}^2 + \frac{1}{I_{2,2,0}}\sum_{n\neq l}^{N}\lambda_{\ln} A_n^2 J_{\ln} \\ & - \mathrm{bB}^2\left(4\frac{I_{0,2,0}}{I_{2,2,0}}\log A_l - 3\frac{I_{0,2,0}}{I_{2,2,0}} - 4\right) \end{aligned} \tag{15}$$

- the frequency

$$k_l' = 4\mathrm{aC}_l k_l, \tag{16}$$

- for the phase

$$\begin{aligned} \theta_l' \;=\; & -2\alpha A_l^2 B_l + 3\mathrm{aB}_l^4 \frac{I_{0,0,2}}{I_{0,2,0}} + \mathrm{ak}_l - \frac{3}{4I_{0,2,0}}\sum_{n\neq l}^{N}\lambda_{\ln} A_n^2 J_{\ln} \\ & + \frac{b}{2}\left(8\log(A_l) - 7 - 8\frac{I_{2,2,0}}{I_{0,2,0}}\right). \end{aligned} \tag{17}$$

4. Results and discussion

4.1. Super-Gaussian pulses

As far as the super-Gaussian pulse is concerned, one picks $f(\tau) = \exp(-\tau^{2m})$ with $m \geq 1$ where the parameter m checks the degree of edge sharpness. When $m = 1$, one reveals the case of a chirped Gausson, while for larger values of m the pulse gradually becomes square-shaped with sharper leading and trailing edges [1]. The super-Gaussian soliton that is going to be studied in the context of multiple channels is

$$q^{(l)}(z, t) = A_l \exp[B_l(t - t_l)]^m \exp\left[i\left(\frac{C_l}{2}(t - t_l)^2 + k_l(t - t_l) + \theta_l\right)\right], \tag{18}$$

$$1 \le l \le N. \tag{19}$$

The energy of the pulse is given by:

$$E = \sum_{l=1}^{N} \int_{-\infty}^{+\infty} |q^l|^2 \mathrm{dt} = \frac{1}{2}\sqrt{\frac{2\pi}{m}} \sum_{l=1}^{N} \frac{A_l^2}{B_l}. \tag{20}$$

The Euler–Lagrange equation brings about the parameter dynamics given below.

4.1.1. Parameter dynamics

- for the amplitude (A_l)

$$A_l' = \mathrm{aA}_l C_l, \tag{21}$$

- for the temporal position (t_l)

$$t_l' = -(a + \alpha A_l^2), \tag{22}$$

- for the inverse of width (B_l)

$$B_l' = 2\mathrm{aB}_l C_l, \tag{23}$$

- for the chirp pulse (C_l)

$$\begin{aligned} C_l' \; = & -2\alpha A_l^2 B_l^2 k_l \frac{\Gamma\left(\frac{1}{2m}\right)}{2^{\frac{m-1}{m}}\Gamma\left(\frac{3}{2m}\right)} - \frac{2m(2m-1)\mathrm{aB}_l^6}{2^{\frac{2m-2}{m}}} \frac{\Gamma\left(\frac{2m-1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)} + 2\mathrm{aC}^2 + \frac{m}{2^{\frac{4m-3}{2m}}} \sum_{n \neq l}^{N} \lambda_{\mathrm{ln}} A_n^2 J_{\mathrm{ln}} \\ & - \mathrm{bB}^2 \left(\frac{\Gamma\left(\frac{1}{2m}\right)}{2^{\frac{-m-1}{m}}\Gamma\left(\frac{3}{2m}\right)} \log A_l - 3\frac{\Gamma\left(\frac{1}{2m}\right)}{2^{\frac{m-1}{m}}\Gamma\left(\frac{3}{2m}\right)} - 4 \right) \end{aligned} \tag{24}$$

- for the frequency (k_l)

$$k_l' = 4\mathrm{aC}_l k_l, \tag{25}$$

- for the phase (θ_l)

$$\begin{aligned} \theta_l' \; = & -2\alpha A_l^2 B_l + \frac{3\mathrm{am}(2m-1)B_l^4}{2^{\frac{2m-1}{m}}} \frac{\Gamma\left(\frac{2m-1}{2m}\right)}{\Gamma\left(\frac{1}{2m}\right)} + \mathrm{ak}_l - 3m\frac{2^{\frac{1-4m}{2m}}}{\Gamma\left(\frac{1}{2m}\right)} \sum_{n \neq l}^{N} \lambda_{\mathrm{ln}} A_n^2 J_{\mathrm{ln}} \\ & + \frac{b}{2}\left(8\log(A_l) - 7 - 2^{\frac{4m-1}{m}} \frac{\Gamma\left(\frac{3}{2m}\right)}{\Gamma\left(\frac{1}{2m}\right)} \right). \end{aligned} \tag{26}$$

4.2. Super-sech pulses

For the super-sech pulse, one selects $f(\tau) = \mathrm{sech}^m(\tau)$ with $m \ge 1$ where the parameter m checks the degree of edge sharpness. For $m = 1$, one gets the case of a chirped sech-pulse. The super-sech solitons that are going to be studied in the context of multiple channels is

$$q^l(z, t) = A_l \mathrm{sech}^m [B_l(t - t_l)] \exp\left[i\left(\frac{C_l}{2}(t - t_l)^2 + k_l(t - t_l) + \theta_l\right)\right], \tag{27}$$

$$1 \le l \le N. \tag{28}$$

The energy of super-sech soliton is given by:

$$E=\sum_{l=1}^{N}\int_{-\infty}^{+\infty}|q^{l}|^{2}\mathrm{dt}=B\left(m,\frac{1}{2}\right)\sum_{l=1}^{N}\frac{A_l^{2}}{B_l}. \tag{29}$$

The Euler–Lagrange equation leads to the following parameter dynamics

4.2.1. Parameter dynamics

- Amplitude (A_l)

$$A_l'=\mathrm{aA}_lC_l. \tag{30}$$

- Temporal position (t_l)

$$t_l'=-(a+\alpha A_l^{2}). \tag{31}$$

- Width (B_l)

$$B_l'=2\mathrm{aB}_lC_l. \tag{32}$$

- Chirp (C_l)

$$\begin{aligned}C_l' \;=\; &-2\alpha A_l^{2}B_l^{2}k_l\frac{p}{r}-4\mathrm{aB}_l^{6}\frac{s}{r}+2\mathrm{aC}^{2}+\frac{1}{r}\sum_{n\neq l}^{N}\lambda_{\mathrm{ln}}A_n^{2}J_{\mathrm{ln}}\\&-\mathrm{bB}^{2}\left(4\frac{p}{r}\log A_l-3\frac{p}{r}-4\right).\end{aligned} \tag{33}$$

- Frequency (k_l)

$$k_l'=4\mathrm{aC}_lk_l \tag{34}$$

- Phase (θ_l)

$$\begin{aligned}\theta_l' \;=\; &-2\alpha A_l^{2}B_l+3\mathrm{aB}_l^{4}\frac{s}{p}+\mathrm{ak}_l-\frac{3}{4p}\sum_{n\neq l}^{N}\lambda_{\mathrm{ln}}A_n^{2}J_{\mathrm{ln}}\\&+\frac{b}{2}\left(8\log(A_l)-7-8\frac{r}{p}\right),\end{aligned} \tag{35}$$

with

$$p=\frac{\Gamma(m)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(m+\frac{1}{2}\right)}, \tag{36}$$

$$r=\frac{2^{-1+2m}}{m^{3}}\,{}_4F_3(m,m,m,2m;1+m,1+m,1+m,-1), \tag{37}$$

$$\begin{aligned}s \;=\; &-\frac{m^{3}p}{2m+1}+\frac{2^{2m-2}m^{2}}{2m+1}\frac{\{\Gamma(m)\}^{2}}{\Gamma(2m)}\\&+\frac{m^{2}(2^{2m+2}+1)}{2(m+2)}\,{}_2F_1(2+m,2+2m;3+m,-1),\end{aligned} \tag{38}$$

where the generalized hypergeometric function is written as:

$${}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;x)=\sum_{k=0}^{\infty}\frac{(a_1)_k(a_2)_k\ldots(a_p)_k}{(b_1)_k(b_2)_k\ldots(b_q)_k}\frac{x^{k}}{k!}; \tag{39}$$

and $(a)_k$ are Pochhammer symbols which mean:

$$(a)_k=a(a+1)\ldots(a+k-1)=\frac{(a+k-1)!}{(a-1)!}. \tag{40}$$

5. Conclusions

In this paper, the dynamics of super-Gaussian and super-sech solitons pulses that propagate through optical fibers have been exhaustively studied. The variational principle was applied to obtain the soliton parameter dynamics of the super-Gaussian and super-sech pulses. These soliton parameter dynamics can be employed to conduct studies in collision induced timing and frequency jitter in solitons, four-wave mixing, ghost pulses, stochastic perturbation of solitons and many more aspects. Additional forms of fiber nonlinearities are also going to be touched upon later with time. These results are all going to be surely and sequentially reported.

Conflict of interest

The authors also declare that there is no conflict of interest.

Acknowledgements

The research work of the sixth author (MRB) was supported by the grant NPRP 11S-1126-170033 fromQNRFand he is thankful for it.

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