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Analysis of the radial flow from a Ground water reservoir to its production well, in steady-state flow conditions, in Monzoungoudo, Benin.

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ABSTRACT

Background: Analyze the groundwater flow is generally to predict the aquifer piezometry under various groundwater stress situations. This paper aimsto analyzetheradial flow, between the groundwater reservoir and its production well, which consequence is the appearance ofthe drawdown curve(cone of depression),in the potentiometric surface caused by the pumping well.When a well is producing, water levels in its neighborhood are lowered. After a certain producing time, with a constant flow, the drawdown cone does not evolve. In this case, the permanentregime of groundwater flow is reached and do not change. It means that the pressure and the fluidvelocity distributions are independent of time; therefore,we consider that the drawdown curve (cone of depression) in the potentiometric surface is timeless in the permanent regime.C. V. Theis developed in 1935, an approach of solution which investigate the behavior of the lowering of the piezometric surface cone generated by the discharge of the pumping well using the groundwater storage.This study, which result presented a simple analytical model behind the calculation of the drawdown cone (cone of depression) in the potentiometric surface, is a refinements of the so-called Theis equation.(See figure n°1).**Objective:** Determination and calculation of the static pressure; the determination of mathematical expression and the calculation of the drawdown curve in the potentiometric surface.**Results:**the expression of static pressure of the reservoir is established and calculated.The mathematical expression of the discharge of hydraulic charge (the drawdown cone in the potentiometric surface) from the reservoir to the well is established.The drawdown curve in the potentiometric surface is drawn.**Conclusion:**The mathematical model, developed to analyze the groundwater flow between the producing well and the reservoir, showed that the fall of the hydraulic load occurs essentially in the vicinity of the producing well**,**and refine the equationofC. V. Theis. It permits to watch the groundwater storage, to control the piezometric lowering on the catching fields or the draining or drying excavations, to do the diagnostics in water borings, or oil or gas drillings.

INTRODUCTION

When groundwater flows from the reservoir to the environment of the producing well by radial flow , the hydraulic charge in the reservoir varies, from the hydraulic chargeh_R, due to the well-range R to the hydraulic chargeh_w, dueto the radius of the production well (a).This study, first develops the theory which proposed the calculation of the drawdown cone (cone of depression) in the potentiometric surface around a potential well, basing on the empirical law, Darcy's law that set the foundation for the analysis of groundwater flow. Theflow regime is a groundwater steady-state flow if the pressure at any point of the reservoir remains constant i.e. does

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not change with the time: $\left(\frac{\partial p}{\partial t}\right) = 0$;and the fluid velocity is Independent of the time.The measureddataateach time, which is well-range-depended variable, isthe drawdown curve(cone of depression), in the potentiometric surface created by the hydraulic charge (in the well). It is a fundamental hydrodynamic parameter for assessing the behavior of the aquifer and for the analysis of the radial flow.

Theradial waterflow from the groundwater reservoir to the production welldepends of thedischarge between the static pressure in the reservoir and the pressure at the bottom of the well.Through this analysis, a mathematical model is developed to determine first, the expression of the static pressure that had been expressed in function of the pressure head of the production well, secondly the analytical expression of the hydraulic charge that define the variation of the cone of the drawdown in the potentiometric surface., difference between the static and pumping water level and in other way, to calculate first the static pressure and secondly the different values of the hydraulic charge in function of the radial distance (r), with a $\leq$ r $\leq$R.
This paper allows in other way, to refine the expression of the equation of Theis.

Nomenclature:

a : radius of the well
r : radial distance a $\leq$ r $\leq$R
R : well-range
h_R : hydraulic charge in the reservoir
h_W : hydraulic charge in the well
p :pressure
p_{st} :static pressure
P_w : pressure at the bottom of the well
h : hydraulic charge
ρ : density of the fluid
g : acceleration of the gravity
z_h : altitude
K :hydraulic conductivity of the medium
μ :dynamic viscosity of the fluid
ϑ : kinematic viscosity of the fluid
k :intrinsic permeability of the medium
ϕ : porosity of the medium
$\vec{v}$: speed of Darcy
S : storage coefficient specific
P_{hq}: hydrodynamic flow pressure,
Δ : pumping-out (drawdown)
NP :piezometric level or groundwater level
H **:** depth of the well
e **:**productive thickness
Q : flow rate
λ :coefficient of resistance

Position of the problem:

The objective of the present paper is to determinate, first the static pressure of the reservoir, function of the pressure at the bottom of the welland then simulatethe variation of the dynamic charge of the groundwater flowing from the well-range R around the production well, to the environment of the well at the a radiusof the production well, and finally to simulate mathematically the potentiometric surface of the depression cone createdby the variation of the well-range, which describes the different levels attended from the static pressure to the hydraulic pressure at the bottom of the well. The figure 1 shows the steady-state flow from a reservoir to a production well.

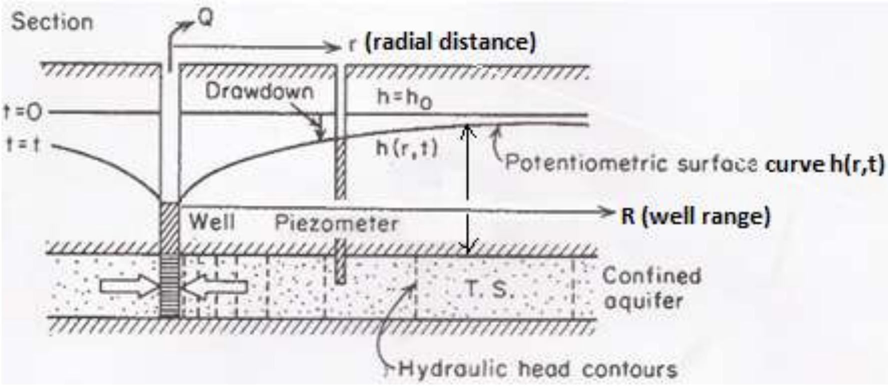

Fig. 1: Steady-state radial flow from a reservoir to a production well

Thephysical model of the problem, is constituted by the reservoir, a porous medium which thickness is$\boldsymbol{e}$; andthe producing wellwhichflowrateis Q. The production well is simulated to a circular pipe which radiusis $\boldsymbol{a}$. Let us consider a cylindrical coordinatessystem (O,r,z),

$h_R$is the hydraulic charge at the limit of the well-range$\boldsymbol{R}$ inthe reservoir, and h_W, the hydraulic charge of the well.

The well-range is the locus point for the well-range R where,the pressuredo not influence the potentio-metric surface inthe reservoir during its exploitation,comparing to its axis, and the value of the depression cone (pumping-out), ie the drawdown is not available; $(h_R - h_w) = 0$

Thus , the water depth we measured in the well,between the production well and the reservoir is the differentvalues of $h(r)$ of the depression cone, with the following approximation:

According to Sichart,$100a < R < 300a with R = 3000(h_R - h_W)\sqrt{K}$

Whereh_R :the reservoir charge;

h_W : the well charge ;

$h_R - h_W$: thepumping depth;

K :the hydraulic conductivity.

In steady-state condition, we get: $R = 1{,}5\sqrt{\frac{K.h_R\ .t}{\phi}}$,where$t$: duration of transition regime and,ϕ the reservoir porosity. The figure 2 shows the variation of the hydraulic charge$h(r)$ in function of the radial distance.

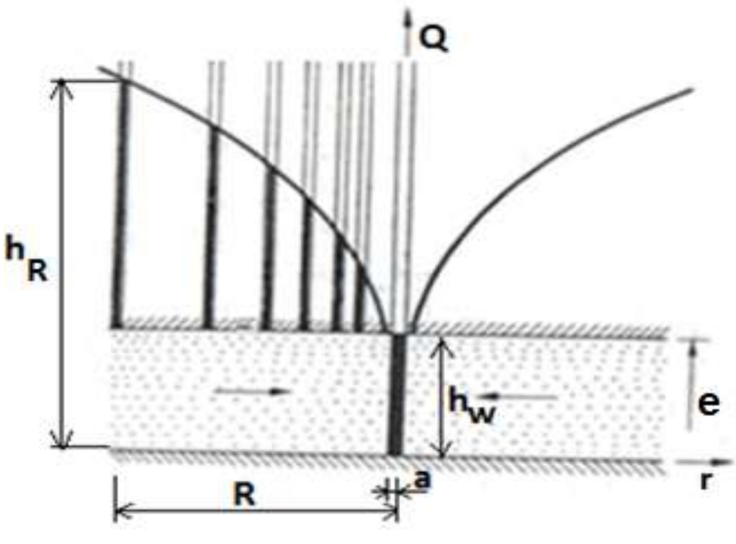

Fig.2: Variation of the drawdown curve in the potentiometric surface of the hydraulic charge from$h_R$toh_w in function of the radial distance(a ≤ r ≤ R)

MATERIALS AND METHOD

The physical model considered here, consisted of a mathematical model flow around the production well constituted by the main hydrodynamic equations, which are the law of Darcy, the continuity equation, and the diffusivity equation, under the following approximations: the porous medium is supposed to be isotropic, homogenous; its permeability,porosityϕ, hydraulic conductivity K, and its productive thickness$e$, are supposed to be constant.

In other side, the densityρ, the dynamic viscosity μ and the cinematic viscosity $\vartheta$of the groundwaterare considered as constants. The water is produced in accordance to the figure 1; a well-known geometry wells. Its diameter d, depth $H$and flow rate$Q$.

Geological and Hydrogeological Setting:

The hydro-geological data used for this study mainly concern the technical data of the drilling carried out in Monzoungoudo village (table1) and which are provided by DGEAU, hydraulic services of Benin.

The main characteristics sought are: transitivity, dynamic viscosity, kinematic viscosity, flow rate, diameter of the well, depth of the well, thickness of the reservoir, density of the fluid, acceleration of the gravity.

Table 1: The Groundwater ReservoirandWell data (DGeau Mai 2000)

VALUES OF CHARACTERISTICS OF RESERVOIR	
Permeability k (Darcy)	21
Depth of the well H (m)	244,18
Thickness of the reservoir e (cm)	4318
Diameter of the well (m)	0, 126
Flow rate Q (cm^3/s)	2000
Density of the fluid ρ (Kg/m^3)	1000
Acceleration of the gravity g (m/S^2 ou N/Kg)	9,81
dynamic viscosity μ (Centipoise)	0,89
Kinematic viscosity ϑ (m^2/S)	$0{,}89.10^{-6}$
Pressure at the head of the well p_2(bars)	4,16

Calculation of the static pressure in the reservoir:

The static pressure of the fluid at any pointof the reservoir is defined, according to the equation of Bernoulli, and Codo, F.P. et al.(2012), at the head of the well, the pressure is:

$p_2 = p_1 - \rho g H - \lambda \rho H \frac{v^2}{2d}$andby transformations

$$p_2 = p_{st} - \rho g H - \frac{\mu Q}{2\pi e k} \ ln\left(\frac{R}{a}\right) - 0{,}06642\rho \frac{H}{d^{4,8}} \ Q^{1,8}\vartheta^{0,2} \quad (1)$$

with

$$p_1 = p_{st} - \frac{\mu Q}{2\pi e k} \ ln\left(\frac{R}{a}\right) \quad ; \quad \lambda = \frac{0{,}086}{Re^{0,2}} \ ; \ R_e = \frac{v \ d}{\vartheta} \quad and \quad v = \frac{4Q}{\pi d^2}$$

Withthe approximation Laurent, H. et al. (1972): $ln(\frac{R}{a}) = 2\pi$,we obtained the static pressure in the reservoir:

$$p_{st} = p_2 + \rho g H + \frac{\mu Q}{ek} \ + 0{,}06642\rho \frac{H}{d^{4,8}} \ Q^{1,8}v^{0,2} \quad (2)$$

The calculation of the static pressure, assumed the use of the parameters of the table1.

Governing Equations:

The boundary conditions for the governing equations are the following :to analyze of the radial flow in steady-state flow conditions, we supposed the following simplifying assumptions:

- the fluid is incompressible ($\rho = cste$),
- flow around the well is radial,
- the porous medium is homogeneous and permeable,
- the porous medium is isotropic ($k_h = k_v = k$),
- the flow is tow-dimensional in the plane (x, y) and ($v_z = 0$),
- the flow is steady ($\frac{\partial}{\partial t} = 0$),
-the flow is conservative ($Q = const$).

The part of the object of our study tank for radius ***R*** around the radius of well ***a***and occupies a limited area recess.

In general, two boundary conditions are associated to the problem:

-At the tank: $r = R$, the charge ish_R

-At the well: $r = a$, the charge is h_w

Used equations:

The hydraulic charge$h$for an incompressible fluid is:

$$h = \frac{v^2}{2g} + \frac{p}{\rho g} + z_h \quad (3)$$

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If the flow in the porous medium is neglected ($v = 0$;) thus, $\frac{v^2}{2g}$ =0 and then the hydraulic charge of the fluidis equivalent to the static charge:

$$h = \frac{p}{\rho g} + z_h \quad (4)$$

The generalized law of Darcy:

The filtration speed is proportional to the gradient of pressure p ; in the case of an incompressible fluid, this law can be expressed in terms the hydraulic charge:

$$\vec{v} = -K\vec{\nabla}h \text{ with } K = \frac{k\rho g}{\mu} \quad \text{and } h = z_h + \frac{p}{\rho g} \quad (5)$$

The continuity equation:

The equation of continuity can be expressed as follows:

$$\frac{\partial}{\partial t}(\phi) + div(\vec{v}) = 0 \quad (6)$$

The diffusivity equation:

The association of the equation of continuity and the law of Darcy,give the diffusivity equation:

$$\frac{\partial(\phi)}{\partial t} + div(-K\vec{\nabla}h) = 0 \quad (7)$$

Considering the porosity ϕindependent of time, the porous medium non-deformable, taking only the hydrodynamic part and the temperature constant,ρdepending onlyon$p$, we got:

$$\frac{\partial(\phi)}{\partial t} = \frac{\partial(\phi)}{\partial p}\frac{\partial p}{\partial t} = \frac{S}{\rho g}\frac{\partial p}{\partial t} \quad (8)$$

With$S = \rho g \frac{\partial(\phi)}{\partial p}$ the storage specific coefficient which shows the ability of the porous medium to releasing fluid under the effect of increase pressure.By introducing the equation (8) in the continuity equation, we obtain:

$$\frac{S}{\rho g}\frac{\partial p}{\partial t} + div(-K\vec{\nabla}h) = 0 \quad (9)$$

Where$h = \frac{p}{\rho g} + z_h$ and$\frac{\partial h}{\partial t} = \left(\frac{\partial p}{\partial t}\rho g - pg\frac{\partial \rho}{\partial t}\right)\frac{1}{(\rho g)^2}$

Since the fluid is incompressible we can neglect the term$\frac{\partial \rho}{\partial t}$. Comparing to the other terms we got$\frac{\partial p}{\partial t} \approx \rho g \frac{\partial h}{\partial t}$if we replace this approximation in the (9), we got the mass balance equation for an incompressible fluid in a non-deformable medium which general form is:

$$S\frac{\partial h}{\partial t} + div(-K\vec{\nabla}h) = 0 \quad (10)$$

The equation (10) is the diffusivity equation for the confined groundwater.

Since we are in steady flow condition, we have $\frac{\partial h}{\partial t} = 0$ and the diffusivity equation become:

$$div(-K\vec{\nabla}h) = 0 \quad (11)$$

This equation (11) allows to estimate the (hydraulic charge) the potentiometric surface created hydraulic charge expression $h(r)$in function of the hydrogeological parameters. In cylindrical coordinates, the diffusivity equation can be written as follows:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{1}{r^2}\frac{\partial^2 h}{\partial \theta^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (12)$$

As flow is symmetric around the well, then, $\frac{\partial^2 h}{\partial \theta^2} = 0$. Likewise there is no vertical flow and so $\frac{\partial^2 h}{\partial z^2} = 0$.Thus the differential equation (12) is reduced to$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = 0$ and becomes to

$$\frac{d^2 h}{dr^2} + \frac{1}{r}\frac{dh}{dr} = 0 \quad (13)$$

Therefore$\frac{1}{r}\frac{d}{dr}\left(r\frac{dh}{dr}\right) = 0$ (14)

The integration steps are:

$$r\frac{dh}{dr} = C_1$$
$$dh = \frac{C_1}{r}dr$$
$$h = C_1 lnr + C_2$$

Using boundary conditions:

$$r = a, \quad h_w = C_1 lna + C_2$$
$$r = R, \quad h_R = C_1 lnR + C_2$$

with

$$C_1 = \frac{h_R - h_w}{ln(R/a)} \text{and} C_2 = h_w - \frac{h_R - h_w}{ln(R/a)} lna$$

The general solution is then:

$$h = \frac{h_R - h_w}{ln(R/a)} lnr + h_w - \frac{h_R - h_w}{ln(R/a)} lna \text{;thus}$$

$$h = \frac{h_R - h_w}{ln(R/a)} ln(\frac{r}{a}) + h_w \quad (15)$$

The flow into the well is calculated using equation of Darcy and taking into account the previous boundary conditions

$$v = -K\frac{dh}{dr} = -K\frac{h_R - h_w}{ln(R/a)} . \frac{1}{r}$$

The flow rate can be determined by evaluating the flow at any radial distance r and integrating over the flow surface. In taking into account$r = a$, we get:

$$v = -K\frac{h_R - h_w}{ln(R/a)}\frac{1}{a}$$
$$Q = \int_A (v)_{r=a} \, dA = \int_{\theta=0}^{2\pi} -K\frac{h_R - h_w}{ln(R/a)}\frac{1}{a} a d\theta e$$
$$Q = -2\pi Ke\frac{h_R - h_w}{ln(R/a)}$$

Thus

$$h_R - h_w = \frac{Q}{2\pi Ke}\left(ln\frac{R}{a}\right) \quad (16)$$

According to Laurent H., Fabris H. and Gringarten C. (1972), in steady flow, the hydrodynamic flow pressure, is in relationship with the transmissivity of the captive reservoir by the following relation$P_{hq} = \frac{\mu Q}{2\pi ek} ln(\frac{R}{a})$, and they supposed that the quantity $ln(\frac{R}{a}) = 2\pi$can be used in this case.By replacing equation (16) in equation (15) we obtain:

$$h = \frac{Q}{2\pi Ke}\left(\ln\frac{r}{a}\right) + h_w \quad (17)$$

The pressure at the bottom of the well P_w is related to the pressure in the reservoir $P_{st}$by the relation Codo, F.P. (1989):

$P_w = P_{st} - \frac{\mu Q}{2\pi ek} ln\left(\frac{R}{a}\right), with \, ln(\frac{R}{a}) = 2\pi so P_w = P_{st} - \frac{\mu Q}{eK}$.

The value of the hydraulic charge of the well is related to the pressure at the bottom of the well by the relation:$h_w = \frac{P_w}{\rho g}$. By substituting$P_w = P_{st} - \frac{\mu Q}{ek}$ into the previous expression, wegot:$h_w = \frac{P_{st}}{\rho g} - \frac{\mu Q}{ek\rho g}$with$K = \frac{k\rho g}{\mu}$; so the expression of$h_w$becomes$h_w = \frac{P_{st}}{\rho g} - \frac{Q}{eK}$

And by substituting this equation into equation (17) we obtained the expression of the hydraulic charge varies logarithmically with the radial distance.

$$h = \frac{Q}{2\pi Ke}\left(\ln\frac{r}{a}\right) + \frac{P_{st}}{\rho g} - \frac{Q}{eK} \quad (18)$$

so the expression of h becomes

$$h = \frac{Q}{2\pi Ke}\left[\left(\ln\frac{r}{a}\right) - 2\pi\right] + \frac{P_{st}}{\rho g} \quad (19)$$

RESULTS AND DISCUSSIONS

The analytical resolution of the equation (19)solved by the governing equations is computed by using theMatlablanguage. The manometer used is for the measurement of the pressure at the head of the well. The flowchart of figure 3 summarized the different steps of the simulation of the resolution.

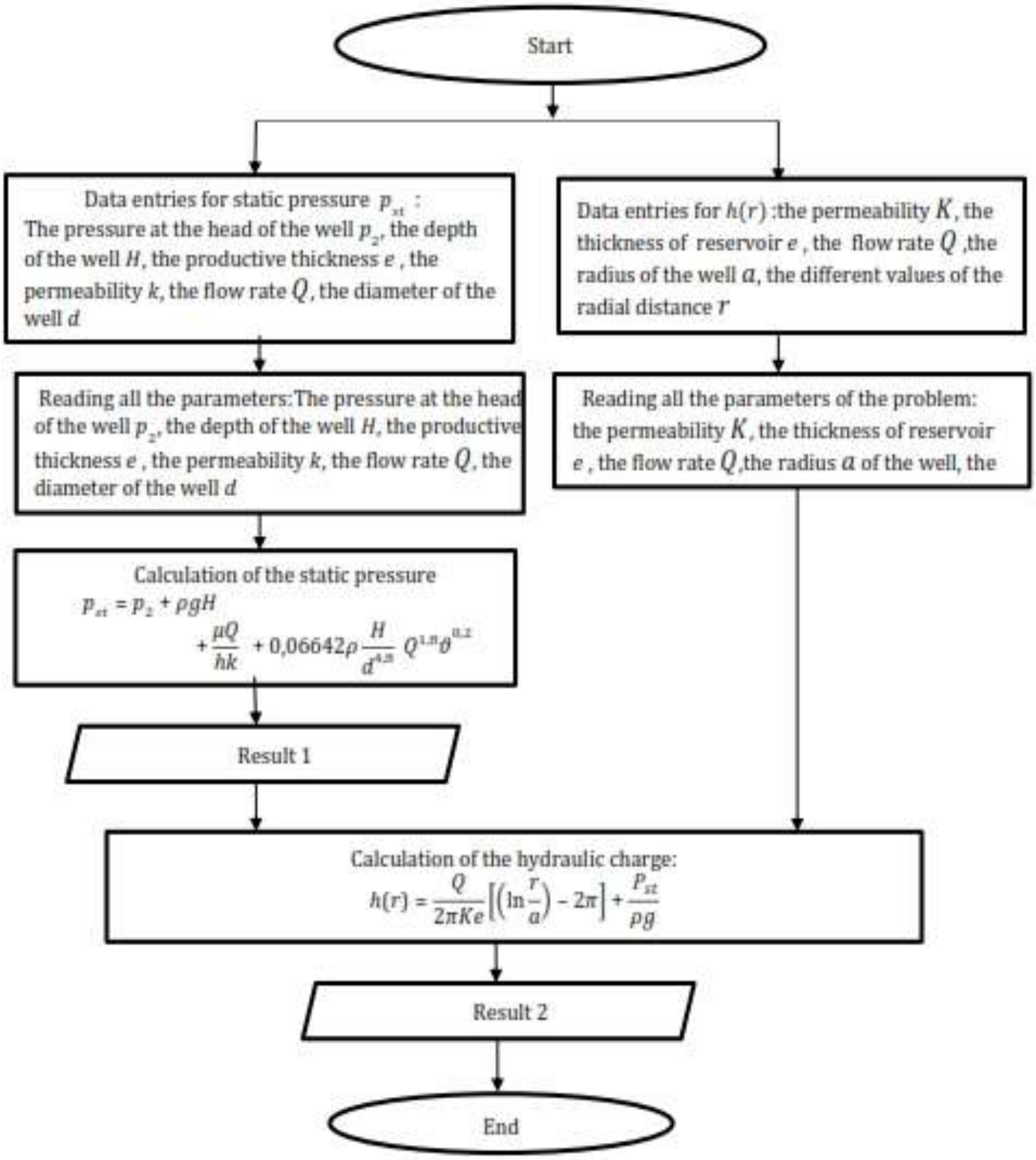

Fig. 3: Organizational chart of the general flow program around the Well

The interface of the program for the radial flow, from a groundwater reservoir of Monzoungoudo to its production well, in steady-state flow conditions is as follows:

Fig. 4: Mode of operation of the program around the well

The results obtained at the end of the program are shown in the following Table 2:

Table 2: Variation of the hydraulic charge as a function of radial distance in the locality of Monzoungoudo

RESULTS OF THE CALCULATION OF THE POTENTIOMETRIC SURFACE CREATED HYDRAULIC CHARGE EXPRESSION$h(r)$											
r (m)	0,063	5	10	15	20	25	30	35	40	45	50
h(m)	266,89	280,79	282,99	284,28	285,20	285,91	286,49	286,98	287,40	287,78	288,11
r (m	55	60	65	70	75	80	85	90	95	100	105
h(m)	288,41	288,69	288,94	289,18	289,4	289,6	289,80	289,98	290,15	290,32	290,47
r (m	110	115	120	125	130	135	140	145	150	155	160
h(m)	290,62	290,76	290,89	291,02	291,15	291,27	291,38	291,50	291,60	291,71	291,81
r (m	165	170	175	180	185	190	195	200	205	210	215
h(m)	291,91	292	292,09	292,27	292,27	292,35	292,44	292,52	292,60	292,67	292,75
r (m	220	225	230	235	240	245	250	255	260	265	270
h(m)	292,82	292,89	292,96	293,03	293,1	293,16	293,23	293,29	293,35	293,41	293,47
r (m	275	280	285	290	295	300	305	310	315	320	325
h(m)	293,53	293,59	293,64	293,70	293,75	293,81	293,86	293,91	293,96	294,01	294,06
r (m	330	335	340	345	350	355	360	365	370	375	380
h(m)	294,11	294,16	294,20	294,25	294,30	294,34	294,39	294,43	294,47	294,52	294,56
r (m)	385	390	395	400	405	410	415	420	425	430	435
h(m)	294,6	294,64	294,68	294,72	294,76	294,8	294,84	294,88	294,92	294,95	294,99
r (m)	440	445	450	455	460	465	470	475	480	485	490
h(m)	295,02	295,06	295,1	295,13	295,17	295,2	295,23	295,27	295,30	295,33	295,37
r (m)	495	500									
h(m)	295,4	295,43									

The following figure 5draws the curve of variation of the potentiometric surface around the well. The variation of the hydraulic load is appreciated as a function of the radial distance.It can therefore be concluded that in thegroundwater reservoir of Monzoungoudo, thehydraulic load charge increases in function of the radial distance with the condition a≤ r ≤R.

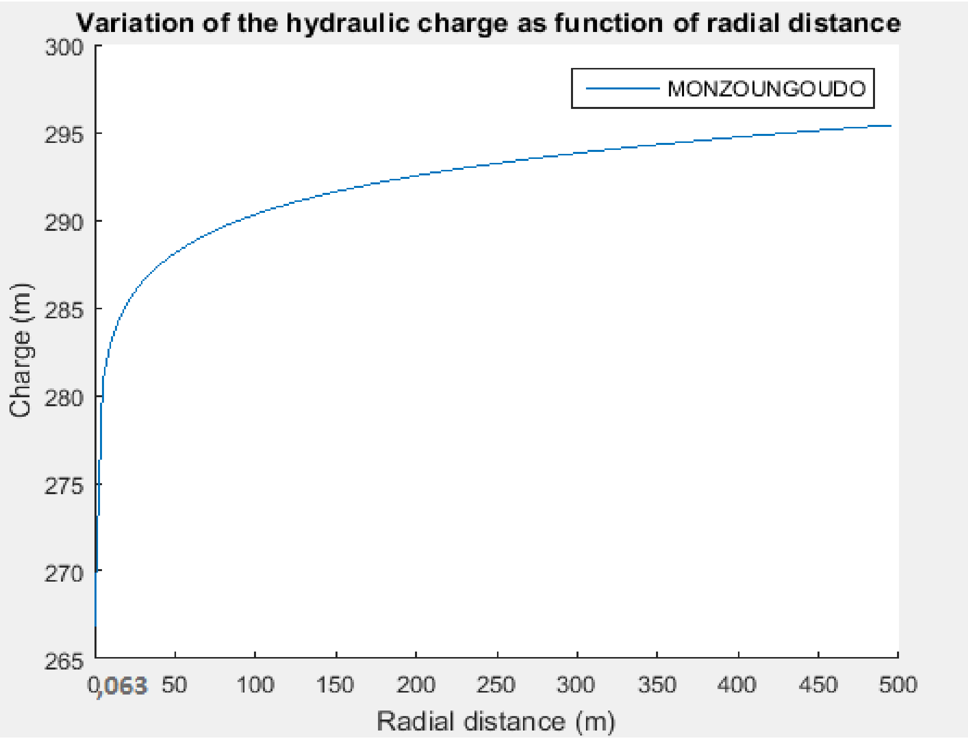

Fig. 5: Variation of the potentiometric surface of the hydraulic charge as a function of the radial distance

Conclusion:

This paper simulated the radial flow between the groundwater reservoir and its production well, taking into account the steady- state flow conditions.

The mathematical model, developed to analyze the groundwater flow between the producing well and the reservoir, is arefinement of the equation of C. V. Theis. The study proved that the fall of the drawdown curve in the potentiometric surface created by hydraulic charge load occurs essentially in the vicinity of the producing well. In one way, it permits to watch the groundwater storage, to control the piezometric lowering on the catching fields or the draining or drying excavations, to do the diagnostics in water borings, or oil or gas drillings and in other way, proved that the hydrodynamic conditions are joined for the ability of the reservoir to produce more water for drinking water needs.

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