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RESEARCH ARTICLE

SOLITON OF SELF-CONSISTENT SYSTEM OF NONLINEAR SPINOR AND GRAVITATIONAL FIELDS WITH THE ARBI- TRARY FUNCTION DEPENDING ON THE BILINEAR PAULI- FIERZ INVARIANT IS = S2 IN GENERAL RELATIVITY

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Abstract

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A self-consistent system of nonlinear spinor and gravitational fields, modeled by static spherical symmetric metric, is considered and studied. Exact spherical symmetric solutions of nonlinear spinor field equations in the Gravitational Theory are obtained. The nonlinearity in the spinor lagrangian is given by an arbitrary function which depends on the invariant generated from the Fierz-Pauli bilinear spinor form IS = S2. It is shown that a soliton-like configuration has a localized energy density and a finite total energy. In addition, The total charge and total spin are also finite. Let us emphasize that the effect of gravitational field on the properties of regular localized solutions significantly depends on the symmetry of the system. The nonlin- ear terms, the gravitational field of elementary particles and the geometrical properties of the metric of the space-time play an important role in the obtaining of analytical solutions having the soliton-like configuration. Let us emphasize that the numerical solutions of the solutions obtained here are presented in graphical form.

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Introduction:-

The description of elementary particles by soliton model in elementary particles physics considering thenonlinearphenomenaandtakingintoaccountthepropergravitationalfieldhasbeenoneofthemostcenturypopulartopics.Notethatthedevelopmentofgeneralrelativity(GR)andquantumfieldtheory(QFT)leads to the increasing interest to study the role of gravitational field in elementary particles physics. This Indeed,thetheorywhichconsiderselementaryparticlesasmaterialpointshasshortcomings.Itisthereasonthatthestudyofanonlinearspinorfieldisveryimportantinordertoovercomethedifficultiesofatheorythatconsiderselementaryparticlesasmathematicalpointsbecausewiththistheoryitisnotpossibletoobtaina finite value of mass, charge and spin of elementary particles. In this approach, elementary particles aremodeled by soliton-like solutions of corresponding to nonlinear equations. In 1995, G.N. Shikin elaboratedthe theory of soliton in General Relativity [1]. The elaboration of this theory leads to flowering of publicationsof articles on the soliton in General

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relativity. Thus, the soliton-like solutions have been obtained in a seriesof remarkable papers [2, 3, 4, 5, 6, 7]. In the same perspective, Arnaud E. Y. and al. determined the exactstatic symmetric soliton-like solutions to nonlinear interacting electromagnetic and scalar field equationstaking into account the own gravitational field of the elementary particles in the plane metric and in thesphericalmetric[8,9,10,11,12].

In this paper we present some regular particle-like solutions in general relativity for describing the configurationofelementaryparticlesinastaticsymmetricmetricdefinedinpseudo-riemannianvarieties.

The paper is organized as follows. In section 2, we write equations for nonlinear spinor fields, the systemof Einstein equations and concepts using the variational principle and usual algebraic manipulations. Section3 addresses the general analytical fundamental solutions. In section 4, we analyzed and discussed in detailthemainresults.Finally,someconclusionsoftheworkareoutlinedinsection5.

Lagrangian,metricandbasicsequations

This section is devoted to the lagrangian, the metric of the space-time and to estabish the fundamental fieldsequations . We shall investigate a self-consistent system of nonlinear spinor and Einstein gravitational fields.Thelagrangianofthetwofieldsisgivenbythefollowingexpression[2]:

$$L=\frac{R}{2\chi}+L_{Sp} \qquad (1)$$

wherethespinorlagragiandensity$L_{Sp}$is

$$L_{Sp}=\frac{i}{2}(\bar{\psi}\gamma^{\mu}\nabla_{\mu}\psi-\nabla_{\mu}\bar{\psi}\gamma^{\mu}\psi)-m\bar{\psi}\psi+L_N . \qquad (2)$$

$L_N$isthenonlineartermofL_{Sp}.Thenonlinearterminthecorrespondinglagrangianmainlydescribestheself-interactionofaspinorfield.$L_N=F(S^2)$isanarbitraryfunctiondependingontheinvariant$S^2=(\bar{\psi}\psi)^2$.

$R=R_{\mu\nu}g^{\mu\nu}$isthe scalar curvature or the traceof Ricci’s tensor.Then,$\chi=\frac{8\Pi G}{c}$is Einstein’s gravitational constant,GisNewton’sgravitationalconstantandcisthespeedoflightinvacuum.ψisthe4-componentsDirac’sspinorwith$\bar{\psi}$ itsconjugate.Inthesequel,weshalldealwiththemetric.

Inthispaper,weconsiderthestaticsphericalsymmetricmetricin[4]:

$$ds^2=e^{2\gamma}dt^2-e^{2\alpha}d\xi^2-e^{2\beta}[d\theta^2+\sin^2(\theta)d\phi^2]. \qquad (3)$$

Forsimplicity,thevelocityoflightistakentobeunity. Wedefinespatialvariableasin[13],$\xi=\frac{1}{\bar{r}}$,where $\bar{r}$ standsfortheradialcomponentofthesphericalsymmetricmetric. Themetricfunctionsα,$\beta$and$\gamma$are stationnary and are functions of ξ only.Thus, they verify the harmonic coordinate condition given by thefollowingexpressionintroducedbyBronnikovK.Ain[13]:

$$\alpha(\xi)=2\beta(\xi)+\gamma(\xi). \qquad (4)$$

Thefieldequationsforthespinorandgravitationalfieldscanbeobtainedfromthevariationalprinciple.Thus,applyingthevariationalprinciplewithrespecttothefunctionψanditsconjugate$\bar{\psi}$,wegetthe nonlinearspinorfieldequationsunderthefollowingform:

$$i\gamma^{\mu}\nabla_{\mu}\psi-m\psi+\frac{dF(S^2)}{dS}\psi=0, \qquad (5)$$

$$i\nabla_\mu\bar{\psi}\gamma^\mu + m\bar{\psi} - \frac{dF(S^2)}{dS}\bar{\psi} = 0, \quad (6)$$

Then, varying the lagrangian (1) with respect to the metric tensor $g_{\mu\nu}$ we obtain the general form of Einstein's equations:

$$G^\nu_\mu = R^\nu_\mu - \frac{1}{2}\delta^\nu_\mu R = -\chi T^\nu_\mu, \quad (7)$$

where G^ν_μ is the Einstein's tensor, R^ν_μ is the Ricci's tensor, δ^ν_μ is the Kronecker's symbol and T^ν_μ is the metric energy-momentum tensor of the spinor field. Then, taking into account (7), we find the components of the tensor G^ν_μ in the metric (3) under the coordinate condition (4) as follpows [4,6]:

$$G^0_0 = e^{-2\alpha}(2\beta'' - 2\gamma'\beta' - \beta'^2) - e^{-2\beta} = -\chi T^0_0, \quad (8)$$

$$G^1_1 = e^{-2\alpha}(2\beta'\gamma' + \beta'^2) - e^{-2\beta} = -\chi T^1_1, \quad (9)$$

$$G^2_2 = e^{-2\alpha}(\beta'' + \gamma'' - 2\beta'\gamma' - \beta'^2) = -\chi T^2_2, \quad (10)$$

$$G^2_2 = G^3_3, \qquad T^2_2 = T^3_3, \quad (11)$$

where prime in previous equations denotes differentiation with respect to variable ξ.

The metric energy-momentum tensor of the material field has the form:

$$T^\nu_\mu = \frac{i}{4} g^{\nu\rho}(\bar{\psi}\gamma_\mu\nabla_\nu\psi + \bar{\psi}\gamma_\nu\nabla_\mu\psi - \nabla_\mu\bar{\psi}\gamma_\nu\psi - \nabla_\nu\bar{\psi}\gamma_\mu\psi) - \delta^\nu_\mu L_{Sp}. \quad (12)$$

By taking into account (5), (6) and (2), the spinor field lagrangian L_{Sp} takes the form:

$$L_{Sp} = \frac{1}{2}\bar{\psi}(i\gamma^\mu\nabla_\mu\psi - m\psi) - \frac{1}{2}(i\nabla_\mu\bar{\psi}\gamma^\mu + m\bar{\psi})\psi + F(S^2), \quad (13)$$

$$= -S\frac{\partial F(S^2)}{\partial S} + F(S^2), \quad (14)$$

$$= -S\frac{\partial F(S^2)}{\partial S} + F(S^2). \quad (15)$$

As the function $\psi = \psi(\xi)$, substituting (15) into (12), we define the nontrivial components of T^ν_μ:

$$T^0_0 = T^2_2 = T^3_3 = -L_{Sp} = S\frac{\partial F(S^2)}{\partial S} - F(S^2), \quad (16)$$

$$T^1_1 = \frac{i}{2}(\bar{\psi}\gamma^1\nabla_1\psi - \nabla_1\bar{\psi}\gamma^1\psi) + S\frac{\partial F(S^2)}{\partial S} - F(S^2). \quad (17)$$

In the precedent equations γ^μ represent Dirac's matrices in curved space-time [13]. They are linked with Dirac's matrices in Minkowski space-time $\bar{\gamma}_a$ as follows:

$$g_{\mu\nu}(\xi) = e^a_\mu(\xi)e^b_\nu(\xi)\eta_{ab}$$

$\mu \quad \nu$

$$\gamma_\mu(\xi)=e^a_\mu(\xi)\bar{\gamma}_a, \qquad (18)$$

where $\eta_{ab}=\mathrm{diag}(1,-1,-1,-1)$ is Minkowski's metric and $e^a(\xi)$ are tetradic 4-vectors.

The relation (18) leads

$$\gamma^0(\xi)=e^\gamma\bar{\gamma}^0,\quad \gamma^1(\xi)=e^\alpha\bar{\gamma}^1,\quad \gamma^2(\xi)=e^\beta\bar{\gamma}^2,\quad \gamma^3(\xi)=\frac{e^{-\beta}\bar{\gamma}^3}{\sin\theta},\quad \gamma^5(\xi)=\bar{\gamma}^5 \qquad (19)$$

The matrices $\bar{\gamma}^\mu$ are chosen as in [14]

$$\bar{\gamma}^0=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix};\quad \bar{\gamma}^1=\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}$$

$$\bar{\gamma}^2=\begin{pmatrix}0&0&0&-i\\0&0&i&0\\0&i&0&0\\-i&0&0&0\end{pmatrix};\quad \bar{\gamma}^3=\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix}$$

$$\gamma^5=\bar{\gamma}^5=\begin{pmatrix}0&0&-1&0\\0&0&0&-1\\-1&0&0&0\\0&-1&0&0\end{pmatrix}$$

In (2), (5)-(6) and (12), ∇_μ has the covariant derivative of the spinor meaning [14,15]. It is linked to the spinor affine connection matrices $\Gamma_\mu(\xi)$ by:

$$\nabla_\mu\psi=\frac{\partial\psi}{\partial\xi^\mu}-\Gamma_\mu\psi \quad \text{or} \quad \nabla_\mu\bar{\psi}=\frac{\partial\bar{\psi}}{\partial\xi^\mu}+\Gamma_\mu\bar{\psi}. \qquad (20)$$

As for Γ_μ, it takes the following general form:

$$\Gamma_\mu(\xi)=\frac{1}{}\,g_{\rho\mu}(\partial_\mu e^b_\sigma e^\rho_a-\Gamma^\rho_{\mu\sigma})\gamma^\delta\gamma^\sigma, \qquad (21)$$

where $\Gamma^\rho_{\mu\sigma}$ are Christoffel's symbols. From (21), we find

$$\Gamma_0=-\frac{1}{}e^{-2\beta}\bar{\gamma}^0\bar{\gamma}^1\gamma',\quad \Gamma_1=0,\quad \Gamma_2=\frac{1}{}e^{-\beta-\gamma}\bar{\gamma}^2\bar{\gamma}^1\beta',\quad \Gamma_3=\frac{1}{}(e^{-\beta-\gamma}\bar{\gamma}^3\bar{\gamma}^1\beta'\sin\theta+\bar{\gamma}^3\bar{\gamma}^2\cos\theta). \qquad (22)$$

Einstein's convention leads

$$\gamma^\mu\Gamma_\mu=\frac{1}{2}(e^{-\alpha}\alpha'\bar{\gamma}^1+\bar{\gamma}^2e^{-\beta}\cot\theta). \qquad (23)$$

By substituting (20) and (23) into (5) and (6), we have

$$ie^{-\alpha}\bar{\gamma}^1\left(\partial_\xi+\frac{1}{2}\alpha'\right)\psi+\frac{i}{2}\bar{\gamma}^2 e\,\psi\cot\theta-(m-S)\psi=0,$$

$$ie^{-\alpha}\bar{\gamma}^1\left(\partial_\xi+\frac{1}{2}\alpha'\right)\psi+\frac{i}{2}\bar{\gamma}^2 e^{\frac{2-\beta}{2}-\alpha}\psi\cot\theta+\left(m-S\frac{dF(S^2)}{d}\right)\psi=0.$$

Considering $\psi(\xi)=V_\delta(\xi)$ with $V_\delta(\xi)=\begin{pmatrix}V_1(\xi)\\ V_2(\xi)\\ V_3(\xi)\\ V_4(\xi)\end{pmatrix}$, for the components of spinor field, we get the following system of equations from (24)

$$V_4'+\frac{1}{2}\alpha' V_4-\frac{i}{2}e^{\frac{\alpha-\beta}{2}}V_4\cot\theta+ie^{\alpha}\left(m-S\frac{dF(S^2)}{d}\right)V_1=0, \tag{26}$$

$$V_3'+\frac{1}{2}\alpha' V_3+\frac{i}{2}e^{\frac{\alpha-\beta}{2}}V_3\cot\theta+ie^{\alpha}\left(m-S\frac{dF(S^2)}{d}\right)V_2=0, \tag{27}$$

$$V_2'+\frac{1}{2}\alpha' V_2-\frac{i}{2}e^{\frac{\alpha-\beta}{2}}V_2\cot\theta-ie^{\alpha}\left(m-S\frac{dF(S^2)}{d}\right)V_3=0, \tag{28}$$

$$V_1'+\frac{1}{2}\alpha' V_1+\frac{i}{2}e^{\frac{\alpha-\beta}{2}}V_1\cot\theta-ie^{\alpha}\left(m-S\frac{dF(S^2)}{d}\right)V_4=0. \tag{29}$$

The functions V_1, V_2, V_3 and V_4 are connected by the relation

$$V_1^2-V_2^2-V_3^2+V_4^2=\text{cste}. \tag{30}$$

In the section 3, we shall resolve the fundamental fields equations.

1. SOLUTIONS OF FIELDS EQUATIONS

In the preceding section we derived the fundamental equations for nonlinear spinor fields and metric functions. This section aims to carry out the resolution of the fundamental fields equations established in the previous section.

Summing the set of equations (26)-(29), we obtain the first-order differential equation for the invariant function S as follows:

$$\frac{dS}{d\xi}+\alpha'(\xi)S=0. \tag{31}$$

The equation (31) has the evident solution

$$S(\xi)=C_0\exp[-\alpha(\xi)], \qquad C_0=\text{const}. \tag{32}$$

The relation (32) reflects the natural link between the nonlinear spinor field of elementary particles and their proper gravitational field.

Using the spinor field equations (24) and (25), the component T_1^1 of the metric energy-momentum tensor may be rewritten in the form:

$$T_1^1=mS-F(S^2). \tag{33}$$

In the following paragraph, we shall resolve Einstein's equations. To this end, since $T_0^0=T_2^2$, we have

$G^0 - G^2 = 0$.Thereforeweobtainthefollowingequation

0 2

$\beta'' - \gamma'' = e^{2\beta+2\gamma}$. (34)

Thetransformationoftheequation(34)leadstoaLiouvilleequationtypehavingthesolutions[1]:

$\beta(\xi) = {}^{A}(1+{}^{2})\ln$ A G $= 1 + {}^{2}\gamma(\xi)$, $GT^2(h,\xi+\xi_1)$ G (35)

$\gamma(\xi) = {}^{A}\ln$ A , (36) $GT^2(h,\xi+\xi_1)$

AandGareintegrationconstantsandT isafunction.

ThefunctionThasthefollowingform:

h

$T(h,\xi+$

$^1\sinh[h(\xi$
$+\xi_1)],h>0(\xi+\xi_1),h=0$ (37)
$^1\sin[h(\xi+\xi_1)],h<0$
h

$\xi_1)$=wherehand$\xi_1$areintegrationconstants.

The general solutions (42) depend on the arbitrary function $L_N = F(S^2)$. Thus, setting an analytical concreteformofthefunction$F(S^2)$,from(42)wecandetermineexplicitly$S(\xi)$. Then,knowing$S(\xi)$,we canfindoutthemetricfunction$g_{11}(\xi)$from(31).Finally,wecangetthefunctions$g_{00}(\xi)$,$g_{22}(\xi)$and$g_{33}(\xi)$fromtheequality(39).

Consideringtheinvariant$S(\xi) = C_0 e^{-\alpha(\xi)}$,wecanestablishtheregularitypropertiesoftheobtainedsolutions.StudyingthedistributionoftheenergyperunitinvariantvolumeT^0 $_0$ $\overline{-3_g}$,wecanestablishthe localizationpropertiesofthesolutions.

ThefollowingparagraphadressestothedeterminationofthefunctionsV$_\delta(\xi)$. Inthisoptic,wemust

α

solvethesetofequations(26)-(29)inmorecompactformifwepasstothefunctions$W_\delta(\xi) = e^{2}\ V_\delta(\xi)$,with $\delta = 1,2,3,4$[18,19].Wehave:

$'\quad {}^{i}\alpha-\beta \quad {}_{\alpha} \quad dF(S^2)$

$W_4 - {}_2 e \quad W_4\cot\theta + ie \quad \overline{} \quad d \quad m - S \quad W_1 = 0,$ (43)

$'\quad {}^{i}\alpha-\beta \quad {}_{\alpha} \quad dF(S^2)$

$W_3 + {}_2 e \quad W_3\cot\theta + ie \quad \overline{} \quad d \quad m - S \quad W_2 = 0,$ (44)

$'\quad {}^{i}\alpha-\beta \quad {}_{\alpha} \quad dF(S^2)$

$W_2 - {}_2 e \quad W_2\cot\theta - ie \quad \overline{} \quad d \quad m - S \quad W_3 = 0,$ (45)

$'\quad {}^{i}\alpha-\beta \quad {}_{\alpha} \quad dF(S^2)$

$W_1 + {}_2 e \quad \overline{W_1\cot\theta - ie} \quad m - S \quad dS \quad W_4 = 0,$ (46)

where

$' \quad ' \quad 1\, ' \quad 1\alpha$ $-$

$\overline{}$

$W_\rho = (V_\rho + {}_2\alpha V_\rho)e^{2}$

Bysubstituting the expressions (35) and(36) into (4), we getthe metric function α(ξ) as follows:

$\alpha(\xi)=^{A\ 3}+^{2}\ \ln$ A .

G $GT^2(h,\zeta+\zeta_1)$ (38)Finallywedefinetherelationsbetweenthemetricfunctionsα(ξ),β(ξ)andγ(ξ):

2+G G

$\beta(\xi)=$ $\alpha(\xi);$ $\gamma(\xi)=$ $\alpha(\xi).$ (39)

4+ 4+

3G 3G

Equation(9)lookslikethefirstintegraloftheequatons(8)and(10).Itisalsoafirstorderdifferentialequation.Then,introducing(33)and(39)into(9),wehave

(4+3G) h —χ(mS−F(S 2))

$(\alpha')^2=$ 2 $e^{2\alpha}$ i . (40)

2 3 4 − α

2G

e^{4+}

+8G+4 2G

Takingintoaccount$\alpha'=-\frac{1dS}{Sd\xi}$andS(ξ)=$C_0e^{-\alpha(\xi)}$,from(40)weobtain

#

dS C_0(4 + S 4+2G (41)

=±√ — 3G)3G2+8 4+3G —χ(mS−F(S^2))

dξ G+4 C

0

Weobtainthegeneralsolutionsoftheequation(41)asfollows:

dS C_0(4+3G)

S 4+3G $-\chi(mS-F(S^2))$

s =±√ (ξ+ξ) (42)

4+2G 0

$3G^2$+8G+4

. (47)

Withthesetofequations(43)-(46)whereW=$W_\delta(\xi)$letuspasstothesystemofequations dependingon functions of the argument S, i.e.$W_\delta(S) = W_\delta(\xi)$, $S(\xi) = C_0e^{-\alpha(\xi)}$.We obtain for $W_\delta(S)$ the set ofequationsasfollows:

dW_4

dS $-iE(S$ —— $^2)W_4+iK($ $^2)W_1=0,$ (48)

S

dW_3 $+iE(S^2)W$ $+iK(S^2)W$ $=0,$ (49)

$_{3\ 2}$

dS

dW_2 $_2)W_2-iK(S$ $^2)W_3=0,$ (50)

dS $-iE(S$ ——

whe re dW_1 $+iE(S^2)W$ $_1$ $—iK(S_2)W_4=0,$ (51)

dS

$_1C_0$ $2+2G$

$4+3G$ $\cot\theta$

$_S$

$E(S^2)=$;

(52)

2 dS

$d\xi$

$\frac{C_0}{}m-S^{dF(S^2)}$ (53)

$K(S^2)=$ S

dS d

$d\xi$

In sequel, we shall transform the equation (48)-(51) to the second order differential equations.In thisperspective, differentiating the equation (48) and substituting the expression of the function $W_1(S)$ and theexpressionofitsderivativeintotheresult,weobtain:

$''$ $K'(S^2)$ $K'(S^2)E(S^2) - K(S^2)E'(S^2)$

$W_4 - \frac{'}{K(S^2)}W_4 + ^2 \quad E\ (\ ^2_2\ )\ ^2(\quad)\frac{}{K(S^2}$ $W_4=0.$ (54)

Similarlydifferentiatingtheequation(51)andintroducingintotheresulttheexpressionof$W_4(S)$andtheexpressionofitsderivative,weobtainthesecond-orderdifferentialequationforthefunction$W_1(S)$:

$''$ $K'(S^2)$ $K(S^2)E'(S^2) - K'(S^2)E(S^2)$

$W_1 - \frac{'}{K(S^2)}W_1 + ^2 \quad E\ (\ ^2_2\ )\ ^2(\quad)\frac{}{K(S^2}$ $W_1=0.$ (55)

Doingthesameoperatingontheequations(49)-(50),wefindthesecond-orderdifferentialequationsobeyedbythefunctions$W_2(S)$and$W_3(S)$asfollows:

$''$ $K'(S^2)$ $K(S^2)E'(S^2) - K'(S^2)E(S^2)$

$W_3 - \frac{'}{K(S^2)}W_3 + ^2 \quad E\ (\ ^2_2\ )\ ^2(\quad)\frac{}{K(S^2}$ $W_3=0.$ (56)

$''$ $K'(S^2)$ $K'(S^2)E(S^2) - K(S^2)E'(S^2)$

$W_2 - \frac{'}{K(S^2)}W_2 + ^2 \quad E\ (\ ^2_2\ )\ ^2(\quad)\frac{}{K(S^2}$ $W_2=0.$ (57)

Bysumming(54)-(55)andsetting$U=W_1+W_4$,weobtainthefollowingsecond-orderdifferentialequationsofthefunction$U(I_S)$:

$U''(S)-$ $\frac{K'(S^2)}{K(S^2)}$ $U'(S)+ 2\ E^2(S^2)-K^2(S^2)U(S)=0.$ (58)

Theequation(58)maybetransformedto: $\frac{}{K(S^2}$ $\frac{}{2\varepsilon dS}$ $\sqrt{}$

1 d

$U'(S)$ $-U(S)=0$ (59) $\frac{}{K(S^2)}$ $\sqrt{}$ 2ε

undertheconditionE^2(S^2)=(1−ε)K^2(S^2)with0<ε≤1[4].

Theequation(59)possessesthefirstintegral

$U'(S)=\pm\ U^2(S)+C_1K(S^2)\sqrt{}\ 2\varepsilon,$ $C_1=\text{const.}$ (60)

ItmaybeclearlyseenthatthegeneralsolutionsofEq.(60)dependonthesignoftheconstantC$_1$.
IfC$_1$=a$^2_1$>0,thentheequation(60)hasthesolution

$U(S)=a_1\sinh N_1(S).$ (61)

IfC$_1$=−b^2<0,thesolutionoftheequationof(57)isgivenby:
$U(S)=b_1\cosh N_1(S).$ (62)

with $N_1(S)=\sqrt{}\ 2\varepsilon\int K(S^2)dS+R_1,$ $R_1=\text{const.}$ (63)

Thedifferenceofequations(48)and(51),takingintoaccountof(61)and(62),gives:

$X(I_S$ $)=$ W_1 $-W_4$ $=-ia_1$ $\sqrt{}$ $1\ \ \varepsilon\ \ 1$ 1 2ε $\sqrt{}$ $\cosh N(S),$ (64)

or

$\sqrt{}\ 1-\varepsilon-1$

$X(S)=W_1-W_4=-ib_1$ $\sqrt{}$ 2ε $\sinh N_1(S),$ (65)

wherea$_1$andb$_1$areintegrationconstants.
Solvinganalogouslytheequations(56)and(57),weobtainthefollowingexpressionsforY(S)=W$_2$+W$_3$asfollows:

$Y(S)=a_2\sinh N_2(S),$ for $C_2=a^2_2>0$ (66)

or

$Y(S)=b_2\cosh N_2(S),$ for $C_2=-b^2_2<0.$ (67)

Intheseconditions,itthenfollowsfromtheexpressions(66)and(67)that:

$V(S)=W_2$ $-W_3$ $=ia_2$ $\sqrt{}$ $1\ \ \varepsilon\ \ 1$ 2 2ε $\sqrt{}$ $\cosh N(S),$ (68)

or

$V(S)=W_2-W_3=ib_2$ $\sqrt{}\ 1-\varepsilon-1$ $\frac{\sqrt{}}{2\varepsilon}$ $\sinh N_2(S),$

$N_2(S)=\sqrt{}\ 2\varepsilon\int K(S^2)dS+R_2,$ (70)

wherea$_2$,b$_2$andR$_2$areintegrationconstants.
Considering the cases whereC$_1$= a$^2_1$>0and C$_2$= −b$^2_2$<0,let us determine the expressions ofthe functionsW$_\delta$(S).WegetforthefunctionsW$_\delta$(S)thefollowingexpressions:

1 1

$$W_1(S)=a_0\sinh N(S)-i\sqrt{\frac{1-\varepsilon^{-1}}{2\varepsilon}}\cosh N(S), \qquad (71)$$

$$W_2(S)=b_0\cosh N(S)+i\sqrt{\frac{1-\varepsilon^{-1}}{2\varepsilon}}\sinh N(S), \qquad (72)$$

$$W_3(S)=b_0\cosh N(S)-i\sqrt{\frac{1-\varepsilon^{-1}}{2\varepsilon}}\sinh N(S), \qquad (73)$$

$$W_4(S)=a_0\sinh N(S)+i\sqrt{\frac{1-\varepsilon^{-1}}{2\varepsilon}}\cosh N(S), \qquad (74)$$

with $a_0=\frac{1}{2}a_1$ and $b_0=\frac{1}{2}b_2$.

Letusnotethatwecanalsoobtaintheexpressionsofthefunstions $W_\delta(S)$ bychoosing $C_1=b_2^2<0$ and $C_2=a_1^2>0$. In addition, in the expressions (63) and (70), we can use the minus sign before the integral.By doing so, we don't loss of generality [17]. We pass to the functions $V_\delta(\xi)$ by multiplying the functions $W_\delta(\xi)$ obtainedintheexpressions(71)-(74)by $e^{-\frac{1}{2}\alpha(\xi)}$ asfollows:

$$V_1(\xi)=a_1\sinh N(\xi)-i\sqrt{\frac{1-\varepsilon^{-1}}{2\varepsilon}}\cosh N(\xi)\exp\left(-\frac{A}{3G}+\frac{2}{1}\ln\frac{A}{GT^2(h,\xi+\xi_1)}\right) \qquad (75)$$

$$V_2(\xi)=b_2\cosh N(\xi)+i\sqrt{\frac{1-\varepsilon^{-1}}{2\varepsilon}}\sinh N(\xi)\exp\left(-\frac{A}{3G}+2\ln\frac{A}{GT^2(h,\xi+\xi_1)}\right) \qquad (76)$$

$$V_3(\xi)=b_2\cosh N(\xi)-i\sqrt{\frac{1-\varepsilon^{-1}}{2\varepsilon}}\sinh N(\xi)\exp\left(-\frac{A}{3G}+2\ln\frac{A}{GT^2(h,\xi+\xi_1)}\right) \qquad (77)$$

$$V_4(\xi)=a_1\sinh N(\xi)+i\sqrt{\frac{1-\varepsilon^{-1}}{2\varepsilon}}\cosh N(\xi)\exp\left(-\frac{A}{3G}+\frac{2}{1}\ln\frac{A}{GT^2(h,\xi+\xi_1)}\right) \qquad (78)$$

Thefollowingsectiondealswiththeanalysisofthegeneralresultsobtainedpreviouslybyconsideringtheconcretenon lineartermsofthearbitraryfunction $F(S^2)$ inthelagrangiandensity.

2. DISCUSSION

In the prsent section we derived the fundamental equations for nonlinear spinor fields and metric functions.This section is devolted to analyze and discuss the main results.Beside this, we have ploted the metricfunctions, the energy density, the energy density per unit invariant volume. By doing so, we have chosen theconcreteformofthearbitraryfunctionundertheform:

$$F(S^2)=\lambda S^2 \qquad (79)$$

where λ isnonlinearityparameter.From(24),wehavethefollowingequation:

$$-\alpha\ 1\ \ ie\ \frac{1}{i}\ \ '\ \ 2-\beta\ \ \frac{dF(S^2)}{}$$

d $\overline{\gamma}(\partial_{\xi}+{}_{2}\alpha)\psi+{}_{2}\overline{\gamma}e$ $\psi\cot\theta-$ $m-S$ $\psi=0.$

Inordertosolvethisaboveequationweneedtogobacktotherelation(42).BysubstitutingF(S^2)=λS^2 into(42)andassumingthat$^{4+2G}_{4+3}$≅1[4],withoutlossofgenerality,weobtain:

$1 - m\chi$!

$\sqrt{}$ $\chi\lambda(4+3G)$ —

$S(\xi)=$ $\frac{C_0}{}$ $-1+\cosh\sqrt{3G^2+8G+4}(\xi+\xi_0)$ (

$2\lambda\chi$

From(32),wededucetheexpressionofthemetricfunctiong$_{11}$α(ξ).Then,from(39)wededucetheexpressionsofthefunctionsβ(ξ)andγ(ξ):

$g11=-e^{2\alpha(\xi)}=-$ $C0^{2}$ $=$ $-$ $1-$ $m\chi$ ^{2}

S $C0$

$2\lambda\chi$

$C0$

$\sqrt{}\chi\lambda(4+3G)$ i (82)

$-1+\cosh\sqrt{}$ $(\xi+\xi\ )$ $\overline{3G^2+8G+}$ 0

$$g_{22} = -e^{2\beta(\xi)} = \frac{C_0}{S} = \frac{C_0}{\left(\frac{1}{C_0}-m\chi\right)\left[-1+\cosh\sqrt{\frac{\chi\lambda(4+3G)}{3G^2+8G+4}}(\xi+\xi_0)\right]} \tag{83}$$

$$g_{33} = g_{22}\sin\theta = \frac{C_0}{S}\sin\theta = \frac{C_0}{\left(\frac{1}{C_0}-m\chi\right)\left[-1+\cosh\sqrt{\frac{\chi\lambda(4+3G)}{3G^2+8G+4}}(\xi+\xi_0)\right]}\sin\theta \tag{84}$$

$$g_{00} = -\left(\frac{C_0}{S}\right)^{\frac{2G}{4+3G}} = -\left(\frac{C_0}{\left(\frac{1}{C_0}-m\chi\right)\left[-1+\cosh\sqrt{\frac{\chi\lambda(4+3G)}{3G^2+8G+4}}(\xi+\xi_0)\right]}\right)^{\frac{2G}{4+3G}} \tag{85}$$

Introducing (82) into (16), the energy density is defined as follows

$$T_0^0(\xi)=\lambda\left(\frac{\frac{1}{C_0}-m\chi}{2\lambda\chi}\right)^2\left[-1+\cosh\sqrt{\frac{\chi\lambda(4+3G)}{3G^2+8G+4}}(\xi+\xi_0)\right]^2 \qquad ($$

Let us note that the energy density is bounded when ξ $[0,\xi_c]$. In virtue of (86), the energy density per unit invariant volume $f(\xi)=T^0_0(\xi)e^{2\alpha-\gamma}\sin\theta$ is defined in the following way:

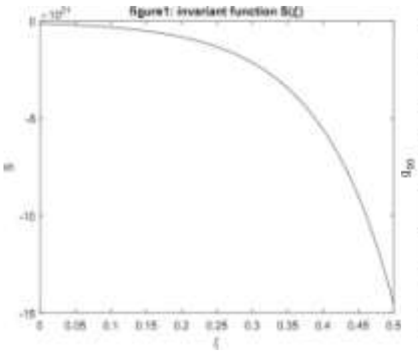
$$f(\xi)=\lambda\left(\frac{\frac{1}{C_0}-m\chi}{2\lambda\chi}\right)^2\left[-1+\cosh\sqrt{\frac{\chi\lambda(4+3G)}{3G^2+8G+4}}(\xi+\xi_0)\right]^2\zeta(\xi)\sin\theta \tag{87}$$

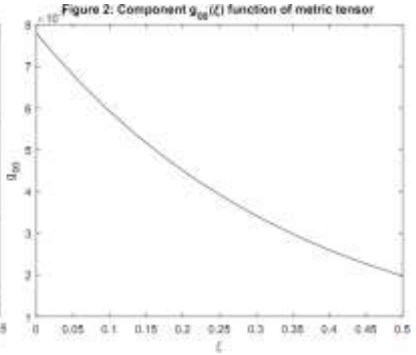
where the function $\zeta(\xi)$ has the form:

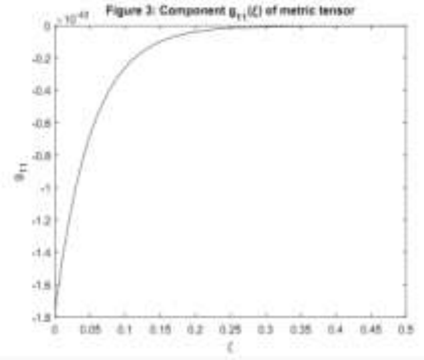
$$\zeta(\xi)=\left(\frac{C_0}{\left(\frac{1}{C_0}-m\chi\right)\left[-1+\cosh\sqrt{\frac{\chi\lambda(4+3G)}{3G^2+8G+4}}(\xi+\xi_0)\right]}\right)^{\frac{8+5G}{4+3G}} \tag{88}$$

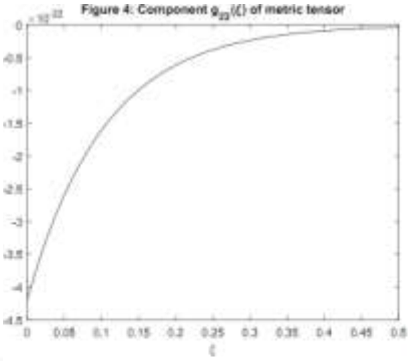
From (88) the energy density per unit invariant volume of Heisenberg-Ivanenko type equation of a non-linear spinor field is localized when $\xi\in[0,\xi]$. Therefore, the total energy $E=\int_0^{\xi_c} f(\xi)d\xi$ is finite.

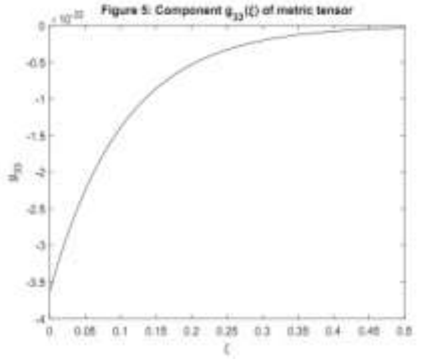
The figues 1 to 7 below, shown numerically the properties of the invariant function $S(\xi)$, non-zero components of the metric $g_{00}(\xi)$, $g_{11}(\xi)$, $g_{22}(\xi)$, $g_{33}(\xi)$ and the energy density T^0_0. Their forms depend on the value of the integration constants.











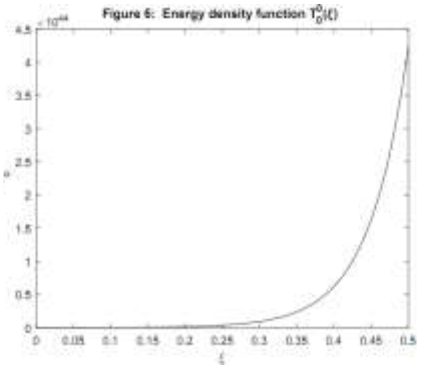


Figure 1 to 5:- Plots of the invariant function $S(\xi)$ and the non-zero components of the metric tensor $g_{00}(\xi)$, $g_{11}(\xi)$, $g_{22}(\xi)$, $g_{33}(\xi)$ for ξ $[0,0,5]$, $\chi=8\pi$, $G=1$, $\lambda=2$, $m=1$, $C_0=0,05$, $\xi_0=5$ and $\theta=\frac{\pi}{3}$. From these graphical illustrations, we note that $S(\xi)$, $g_{00}(\xi)$, $g_{11}(\xi)$, $g_{22}(\xi)$ and $g_{33}(\xi)$ are regular and localized functions. This is in good agreement with the analytical results.

Figure 6: Plot of the energy density $T^0_0(\xi)$ with the same value of constants as previously. Let us emphasize that the energy density $T^0_0(\xi)$ is an asymptotical and localized function.

Let us find an explicit form of the function $V_\delta(\xi)$, $\delta=1,2,3,4$. To this end, we deduce from (63) and (70) the expressions of the functions $N_1(\xi)$ and $N_2(\xi)$ knowing that $F(S^2)=\lambda S^2$ and set $\frac{4+2G}{4+3G}\simeq 1$, without loss of generality:

$$N_{1,2}(\xi)=-2\lambda\sqrt{2\varepsilon(\xi+\xi_0)+M}\tanh\left[\sqrt{\frac{m\chi C_0(4+3G)}{(1+\lambda\chi C_0)(3G^2+8G+4)}}(\xi+\xi_0)+R_{1,2}\right] \quad (89)$$

where $\sqrt{}$ $3\sqrt{}$ 2

$$M=\frac{2\ \ 2\varepsilon(1+\lambda\chi C_0)^2\quad 3G+8G+4}{m\chi^2 C_0(4+3G)}=\text{const.}$$

Then,theexpressionsof$N_1(\xi)$and$N_2(\xi)$aresubstitutedinto(71)-(74)toproducetheexplicitformofthefunctions$W_\delta(\xi)$thatwemultiplybye$^{-\frac{1}{2}\alpha(\xi)}$togetexplicitely$V_\delta(\xi)$asfollows:

$1-\varepsilon-1$ $\quad 1 \quad -\frac{m\chi}{C_0}\ !\ \frac{1}{2}\quad \chi\lambda(4+3G) \quad \frac{1}{2}$

$$V_1(\bar{\xi})=a_0\quad \sinh N_1(\xi)-i\quad \sqrt{}_{2\varepsilon}\quad \cosh N_1(\xi)\quad 2\lambda\chi C_0\quad -1+\cosh\sqrt{3G^2+8G+4}(\xi+\xi_0) \tag{90}$$

$1-\varepsilon-1$ $\quad 1 \quad -\frac{m\chi}{C_0}\ !\ \frac{1}{2}\quad \chi\lambda(4+3G) \quad \frac{1}{2}$

$$V_2(\bar{\xi})=b_0\quad \cosh N_2(\xi)+i\quad \sqrt{}_{2\varepsilon}\quad \sinh N_2(\xi)\quad 2\lambda\chi C_0\quad -1+\cosh\sqrt{3G^2+8G+4}(\xi+\xi_0) \tag{91}$$

$1-\varepsilon-1$ $\quad 1 \quad -\frac{m\chi}{C_0}\ !\ \frac{1}{2}\quad \chi\lambda(4+3G) \quad \frac{1}{2}$

$$V_3(\bar{\xi})=b_0\quad \cosh N_2(\xi)-i\quad \sqrt{}_{2\varepsilon}\quad \sinh N_2(\xi)\quad 2\lambda\chi C_0\quad -1+\cosh\sqrt{3G^2+8G+4}(\xi+\xi_0) \tag{92}$$

$1-\varepsilon-1$ $\quad 1 \quad -\frac{m\chi}{C_0}\ !\ \frac{1}{2}\quad \chi\lambda(4+3G) \quad \frac{1}{2}$

$$V_4(\bar{\xi})=a_0\quad \sinh N_1(\xi)+i\quad \sqrt{}_{2\varepsilon}\quad \cosh N_1(\xi)\quad 2\lambda\chi C_0\quad -1+\cosh\sqrt{3G^2+8G+4}(\xi+\xi_0) \tag{93}$$

Letusemphasizethattheequation(80)hassoliton-likesolutions.Here,theexistenceofthesoliton-like configurations with localized energy density, finite total energy in Heisenberg-Ivanenko type nonlinearequationisaninterestingresult.

Usingthesolutions(90)-(93)wecandterminethecomponentsofthespinorcurrentvector$j^\mu=\bar{\psi}\gamma^\mu\psi$ [16]underthegeneralform

$$j^0=(V_1^*\ V_1+V_2^*\ V_2\qquad +V_3^*\ V_3\qquad +V_4^*\ V_4)e^{-(\alpha+\gamma)}, \tag{94}$$

$$j^1=(V_1^*\ V_4+V_2^*\ V_3+V_3^*\ V_2+V_4^*\ V_1)e^{-2\alpha}, \tag{95}$$

$$j^2=-i(V_1^* V_4-V_2^* V_3+V_3^* V_2-V_4^* V_1)e^{-(\alpha+\beta)}, \qquad (96)$$

$$j^3=(V_1^* V_3-V_2^* V_4+V_3^* V_1-V_4^* V_1)e^{-(\alpha+\beta)}. \qquad (97)$$

In the case of Heisenberg-Ivanenko type nonlinear equation, the components of the spinor current vector may be rewritten in the following way:

$$j^0=2e^{-\alpha-\gamma}\left(a_0^2\left[\sinh^2N_1(\xi)+\frac{-1+\sqrt{1-\varepsilon^2}}{2\varepsilon}\cosh^2N_1(\xi)\right]+b_0^2\left[\cosh^2N_2(\xi)+\frac{-1+\sqrt{1-\varepsilon^2}}{2\varepsilon}\sinh^2N_2(\xi)\right]\right) \qquad (98)$$

$$j^1=2e^{-2\alpha}\left(a_0^2\left[\sinh^2N_1(\xi)-\frac{-1+\sqrt{1-\varepsilon^2}}{2\varepsilon}\cosh^2N_1(\xi)\right]+b_0^2\left[\cosh^2N_2(\xi)-\frac{-1+\sqrt{1-\varepsilon^2}}{2\varepsilon}\sinh^2N_2(\xi)\right]\right) \qquad (99)$$

$$j^2=4e^{-\alpha-\beta}\left(a_0^2\frac{-1+\sqrt{1-\varepsilon^2}}{2\varepsilon}\cosh N_1(\xi)\sinh N_1(\xi)-b_0^2\frac{-1+\sqrt{1-\varepsilon^2}}{2\varepsilon}\cosh N_2(\xi)\sinh N_2(\right. \qquad ($$

$$j^3=0 \qquad (101)$$

As in this study the configuration is static, the components j^1, j^2 and j^3 are evident. But only the component j^0 is nonzero. With this assumption, we get $a_0= b_0= a$, $R_1= R_2= R$, $N_1(\xi) = N_2(\xi) = N(\xi)$ and $\varepsilon = 1$. From the component j^0, we define the charge density or the chronometric invariant of the spinor field as follows:

$$Q(\xi)=j_0\,{}_{0}^{\frac{1}{2}}=4a\,\vartheta(\xi)\cosh 2N(\xi) \qquad (102)$$

where $N(\xi)$ is defined by the expression (89) and

$$\vartheta(\xi)=e^{-\alpha(\xi)}=\frac{C_0}{\frac{1}{-m\chi}\left(-1+\cosh\sqrt{\frac{\chi\lambda(4+3G)}{3G^2+8G+4}}(\xi+\xi_0)\right)}. \qquad (103)$$

The charge density is localized when ξ $[0,\xi_c]$. The total charge of the spinor field in the Heisenberg-Ivanenko type nonlinear equation is:

$$Q=\int_0^{\xi_c}\rho\sqrt{-{}^3g}\,d\xi=4a^2\int_0^{\xi_c}\cosh 2N(\xi)e^{\alpha-\gamma}\sin\theta\,d\xi< \quad , \qquad (104)$$

ξ_C being the center of the field configuration and

$$e^{\alpha-\gamma}=\frac{1}{m\chi}\frac{C_0}{-1+\cosh\sqrt{\frac{\chi\lambda(4+3G)}{3G^2+8G+4}}(\xi+\xi_0)} \qquad (105)$$

$$\frac{C_0}{2\lambda\chi C_0}$$

From (104) the total charge is finite when ξ $[0,\xi_c]$.

Let us deal with the spin tensor of the nonlinear spinor field. Its general form is:

$$S^{\mu\nu,\lambda}=\frac{1}{4}\bar{\psi}\{\gamma^{\lambda}\sigma^{\mu\nu}+\sigma^{\mu\nu}\gamma^{\lambda}\}\psi. \qquad (106)$$

Using(106),thespatialdensityofthespintensor$S^{ik,0}$,i,k=1;2;3is:

$$S^{ik,0}=\frac{1}{4}\bar{\psi}\{\gamma^{0}\sigma^{ik}+\sigma^{ik}\gamma^{0}\}\psi=\frac{1}{2}\bar{\psi}\gamma^{0}\sigma^{ik}\psi. \qquad (107)$$

Thus,wehave

$$S^{12,0}=S^{13,0}=0. \qquad (108)$$

$$S^{23,0}=2a^{2}\cosh 2N(\xi)e^{-\alpha}. \qquad (109)$$

Therelation(109)leadstothedefinitionofthechronometricinvariantofthespatialdensityasfollows:

$$S_{chI}^{23,0}=S^{23,0}=2a^{2}\cosh 2N(\xi)e^{-\alpha}. \qquad (110)$$

Thus,theprojectionofthespinvectorontheradialaxishastheform:

$$S_{1}=2\int_{0}^{\xi_c}S_{chI}^{23,0}\sqrt{-{}^{3}g}\,d\xi=2a^{2}\int_{0}^{\xi_c}\cosh 2N(\xi)e^{\alpha-\gamma}\sin\theta\, d\xi. \qquad (111)$$

Notethatthespintensorofthespinorfieldhasafinitevalueandpositivebecausetheintegrandispositive.

We can conclude that the equation (80) possesses soliton-like configuration with finite value of the totalcharge and the total spin.In addition, the metrics functions are stationnary and regular.Therefore, thesesolutionsmustbeusedtodescribetheconfigurationofelementaryparticleswithmass.

Conclusions:-

In this manuscript,taking into account the proper gravitational field of elementary particles, we obtainedthegeneralsolutionsofEinsteinandnonlinearspinorfieldequations.Weanalyzedinparticularthe Heisenberg-Ivanenko type nonlinear spinor field equations.We note that the solutions of Heisenberg-Ivanenko equation are regular and possess a bounded energy density and limited total energy. Similarly, themetric functions are stationary. The total charge and the total spin are finite quantities as well. We demon-stratedthatthesoliton-likesolutionsexistinflatspace-timeandabsentinlinearcase.Thenonlinearityofthe spinor field vanishes in the space-time without gravitation. Therefore, we note that, the gravitationalfield is nonlinear by nature and its nonlinearity induces the nonlinearity of the spinor field.The numerical solutions of the solutions obtained here are presented in graphical form. We noted that the numerical solutionsare ingoodagreementwiththeanalyticalresults.Solitons ofnonlinearinteractingspinorandscalarfieldsincylindricallysymmetricconfigurationwillbeinthecoreofthe forthcomingpaper.

ConflictsofInterest

Theauthorsdeclarenoconflictsofinterestregardingthepublicationofthispaper.

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