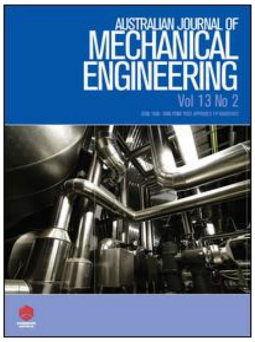

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Australian Journal of Mechanical Engineering

ISSN: 1448-4846 (Print) 2204-2253 (Online) Journal homepage: http://www.tandfonline.com/loi/tmec20

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AUSTRALIAN JOURNAL OF MECHANICAL ENGINEERING
https://doi.org/10.1080/14484846.2018.1523294

Effect of anisotropy in permeability on thermal convection of viscoelastic fluids in rotating porous layer heated from below

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ABSTRACT

In this work, we studied the linear stability analyses of thermal convection, in a viscoelastic fluid saturated rotating anisotropic porous layer heated from below. The influence of hydrodynamic anisotropy and of rotation on the convective phenomenon are considered and investigated by extending the modified Darcy-Maxwell-Jeffrey model to include the Coriolis force term. It is shown that the anisotropic permeability ratio K, the inclination angle of the principal axes (φ) of the porous medium, the relaxation time (ε), the retardation time (ζ) and the Taylor Darcy number (T_D) have a strong influence on the convective flow.

ARTICLE HISTORY
Received 5 January 2018
Accepted 29 August 2018

KEYWORDS
Convection heat transfer; anisotropy; rotating anisotropic porous layer; viscoelastic fluid

1. Introduction

Buoyancy-induced convection in a fluid saturated porous medium is of considerable interest, owing to several geophysical and engineering applications. Prominent among these are

insulation techniques, flows in soils aquifers, petroleum extraction, storage of agricultural products, underground diffusion of contaminants and porous material regenerative heat exchangers. The study of stability of rotating viscoelastic fluid saturated porous layers has received the attention of many researchers in the recent decades. The effect of rotation on the overstability of viscoelastic fluid layer was analysed by Bhatia and Steiner (1972) and they found that rotation has a destabilising influence on the oscillatory convection problem. But this result is in contrast with an ordinary viscous fluid on which rotation has a stabilising effect. Chanderasekhar (1953) investigated the effect of Coriolis force on thermal instability of a fluid layer using various procedures and found that convection sets in for high Rayleigh number.

The onset of buoyancy driven motion in a horizontal porous layer saturated by viscoelastic fluids was analysed by Kim, Lee, and Chung (2003). They found that the onset of convection bifurcation is supercritical and stable. Kumar and Mohan (2012) studied the effect of thermal instability of a heterogeneous oldroydian viscoelastic fluid heated from below in porous medium. They derived the sufficient conditions for non-existence of overstability. A linear stability analysis was carried out to study the onset of convection of viscoelastic fluid in a horizontal fluid layer heated underneath by Rajib and Layek (2012) and they inferred that relaxation time destabilises the system while retardation time stabilises the system.

Ramesh and Arvind (2013) analysed thermal instability of Maxwell viscoelastic fluids in the presence of rotation and variable gravity and found that rotation has a stabilising effect, while variable gravitys field has destabilising effect on the system. The stability of an incompressible simple viscoelastic fluid heated from below has been discussed by Sokolov and Tanner (1972) and they found the stability criteria for the oscillatory mode. Vest and Arpaci (1969) analysed the thermal instability of Maxwell fluid which is heated from below. They confirmed that the elasticity property has a destabilising effect on the liquid which is heated from below and oscillatory convection sets in for low Rayleigh number. Even though most of the earlier literatures apply Darcy law to characterise Newtonian fluids in porous media, few works are also found on modified Darcy law for basic understanding of convection and heat transfer in a non-Newtonian fluid saturated porous medium. Tan and Masuoka (2007) proposed a Darcy Maxwell model by introducing a flow resistance term to overcome the disadvantage encountered in the modified Darcy Maxwell Jeffery model. Yoon, Kim, and Choi (2003) investigated the influence of viscoelastic effects of saturated liquids on the onset conditions in connection with oscillatory instabilities at the threshold of stationary convection. They found that the resulting oscillatory instabilities occur at lower values of Rayleigh – Darcy number than the critical value for the stationary convection. The

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criteria of occurrence of onset conditions of stationary convection on thermal viscoelastic fluids in rotating fluids through porous medium heated from below has been analysed by Saini et al. (2011) and they observed that critical Rayleigh Darcy number for overstability increases with increase in retardation time and Taylor-Darcy number. Sumathi and Aiswarya (2017) studied linear stability analysis of thermal convection in a viscoelastic fluid saturated porous medium subject to uniform rotation. They observed that increasing strain retardation time and stress relaxation time stabilises the system. Also it is observe that due to increase in rotation parameter and Rayleigh-Darcy number R_D the disturbance decay. It can be inferred from this work that the system remains stable when subjected to long waves. Extending the modified Darcy-Maxwell Jeffrey model for a viscoelastic fluid in porous medium to include the Coriolis term, Kang, Fu, and Tan (2011) conducted linear and nonlinear instability analyses of thermal convection in a viscoelastic fluid saturated rotating porous layer heated from below. The effect of rotation is highlighted. The linear analysis results indicate that the critical Rayleigh-Darcy number for overstability increases with increase in retardation time and Taylor-Darcy number, while decreases with increase in relaxation. All these eminent authors have conducted their studies while considering the isotropic porous medium. The work that has been investigated in an isotropic porous medium, over the past two decades, are mostly recorded in a reference book by Nield and Bejan (1992).

Anisotropy, which is generally a consequence of a preferential orientation or asymmetric geometry of the grain or fibres, is in fact encountered in all those applications in industry and nature. In the present paper we consider that the porous medium is homogeneous and anisotropic in permeability with arbitrarily oriented principal axes, as seen in nature and many practical applications. Analytical investigation is conducted for the thermal convection in a viscoelastic fluid saturated porous layer subject to uniform rotation. On the basis of the generalised Darcy-Maxwell-Jeffrey model, the Navier-Stokes equations and the energy equation, the effects of anisotropic parameters of the porous matrix, of the relaxation time (ε), of the retardation time (ζ) and of the Taylor-Darcy number (T_D) on Rayleigh-Darcy number are investigated in detail.

2. Mathematical formulation

The physical model is an infinite horizontal porous layer of thickness h, which is confined between two infinite planes. The porous medium is heated from below with a constant temperature difference $\Delta T = T_2 - .T_1$ across the thickness. Meanwhile, the porous layer subject to rotation with uniform vertical angular velocity ω is saturated with a Maxwell-Jeffrey fluid with constant relaxation time ε^* (which is used in combination with tests pre-setting the strain (deformation) or strain rate (shear rate)) and retardation time ζ^* (which is the delayed response to an applied force or stress and can be described as 'delay of the elasticity') . A Cartesian coordinate frame of reference is chosen to link together with the porous matrix as shown in Figure 1. The upper impermeable surface is exposed to a constant temperature T_1. The lower impermeable plate lining the porous layer is maintained at a constant temperature T_2 ($T_1 < T_2$).

The axial and transverse coordinates are respectively x, z and y, the latter being measured vertically upwards from the lower impermeable wall. The porous medium is anisotropic in flow permeability, the permeabilities along the three principal axes of the porous matrix are denoted by K_1, K_2 and K_3. The anisotropy of the porous layer is characterised by the permeability ratio $K^* = K_1/K_2$, $\gamma = K_1/K_3$ and the orientation angle φ, defined as the angle between

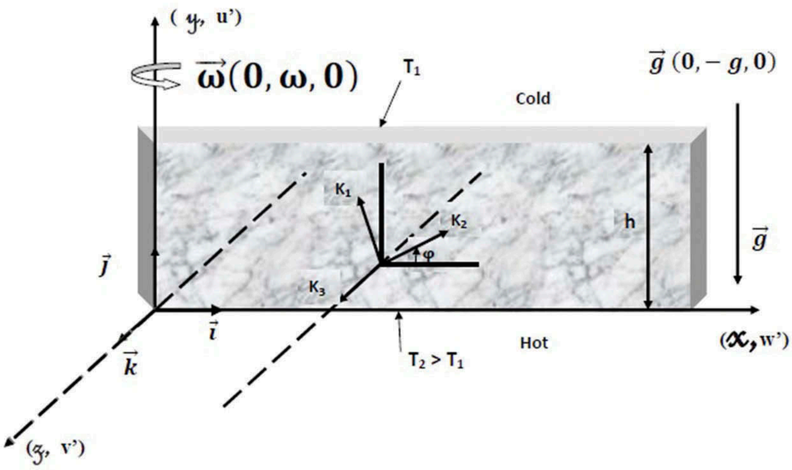

Figure 1. Physical model and coordinate system.

the horizontal direction and the principal axis with the permeability K_2. The porous bed is saturated with a viscous fluid that is in local thermodynamic equilibrium with the solid matrix.

The equations governing the conservation of mass, momentum and energy can be written as follows, to only include the Coriolis force and employing the Boussinesq approximation (Bertola and Cafaro 2006; Kim, Lee, and Chung 2003; Niu, Fu, and Tan 2010):

$$\vec{\nabla} \cdot \vec{V} \;=\; 0, \tag{1}$$

$$\mu(\bar{\bar{K}})^{-1}(1+\zeta^* \frac{\partial}{\partial t})\vec{V} = \left(1+\varepsilon^* \frac{\partial}{\partial t}\right)\left(-\vec{\nabla}P + \rho\vec{g} + \frac{2\rho_0}{\eta}(\vec{\omega}\cdot\vec{V})\right), \tag{2}$$

$$\sigma\frac{\partial T}{\partial t} + (\vec{V}\cdot\vec{\nabla})T = \alpha\nabla^2 T, \tag{3}$$

In these equations, $\vec{V}$ denotes the velocity vector, p the pressure and T the temperature. η the porosity of the porous medium and $\vec{g}$ the gravity acceleration. Moreover, μ is the dynamic viscosity, $\alpha = (k/(\rho c_p))$ is the thermal diffusivity, $\sigma = ((\rho C_p)_f\eta + (1-\eta)(\rho C_p)_s)/(\rho C_p)_f$, where $(\rho c_p)_f$ and $(\rho c_p)_s$ are the volumetric heat capacity of the fluid and the porous medium respectively. In Equation (2), the symmetrical third-order permeability tensor $\bar{\bar{K}}$ is defined as (Yovogan, Degan, and Fagbemi (2018)):

$$\bar{\bar{K}} = \begin{pmatrix} K_1\cos^2\varphi + K_2\sin^2\varphi & (K_2-K_1)\sin\varphi\cos\varphi & 0 \\ (K_2-K_1)\sin\varphi\cos\varphi & K_2\cos^2\varphi + K_1\sin^2\varphi & 0 \\ 0 & 0 & K_3 \end{pmatrix} \tag{4}$$

In accordance with the Boussinesq approximation, the fluid density is assumed to be constant everywhere except in the buoyancy term of the momentum equation where its dependence on temperature is assumed to be linear and is,

$$\rho = \rho_0[1-\beta(T-T_0)]. \tag{5}$$

where ρ_0 denotes the fluid density at the reference temperature T_0 and β the thermal-expansion coefficient.

At quiescent state the temperature varies linearly across the layer thickness, and hence the basic state of the system is given by $\vec{V}_b = \vec{0}$, $T_b = T_2 - \frac{\Delta T}{h}y$, $\frac{dP_b}{dy} + \rho_b g = 0$ with $\rho_b = -\rho_0\beta\frac{\Delta T}{h}y$ and the subscript 'b' denotes the basic state.

For the analysis stability of this system, infinitesimal perturbations should be super- imposed on the basic state in the following form where the primes denote the infinitesimal perturbations:

$$\left\{\begin{array}{rcl} T &=& T_b + \theta'(x',y',z',t'), \\ \vec{V} &=& \vec{V}_b + W'(x',y',z',t'), \\ P &=& P_b + P'(x',y',z',t'). \end{array}\right. \tag{6}$$

the equations governing the perturbation quantities become:

$$\vec{\nabla}\cdot\vec{V}\,' \;=\; 0, \tag{7}$$

$$(\bar{\bar{K}})'\left(1+\zeta^*\frac{\partial}{\partial t}\right)\vec{V}' = \frac{K_1}{\mu}\left(1+\varepsilon^*\frac{\partial}{\partial t}\right)\left(-\vec{\nabla}P' + \rho_0\beta\theta' g\vec{j} - \frac{2\rho_0\omega}{\eta}(\vec{j}\cdot\vec{V}')\right), \tag{8}$$

$$\sigma\frac{\partial\theta'}{\partial t} + (\vec{V}'\cdot\vec{\nabla})\theta' - \frac{\Delta T}{h}W' = \alpha\nabla^2\theta', \tag{9}$$

where u' is the component of $\vec{V}'$ in the y-direction. The governing equations are non-dimensionalised by the following variables:

$$\left.\begin{array}{ll} (x,y,z) = (x',y',z')/h, & (u) = (u')/(\alpha/h), \\ t = t'/(h^2/\alpha), & P = P'/(\alpha\mu^2/h), \\ \zeta = \zeta^*\alpha/h^2, & \varepsilon = \varepsilon^*\alpha/h^2, \sigma = 1. \end{array}\right\} \tag{10}$$

Using these scales, Equations (7)–(9) can be transformed to the following dimensionless form:

$$\vec{\nabla}\cdot\vec{V} \;=\; 0, \tag{11}$$

$$\frac{1}{Da}(\bar{\bar{K}})'\left(1+\zeta\frac{\partial}{\partial t}\right)\vec{V} \;=\; \frac{K_1}{\mu}\left(1+\varepsilon\frac{\partial}{\partial t}\right)\left(-\vec{\nabla}P + Ra\theta\vec{j} - Ta^{1/2}(\vec{j}\cdot\vec{V})\right), \tag{12}$$

$$\frac{\partial\theta}{\partial t} + (\vec{V}\cdot\vec{\nabla})\theta - W = \nabla^2\theta. \tag{13}$$

In the above equations, $Ra = \rho_0 g\beta\Delta Th^3/\mu\alpha$ is the Rayleigh number, $Ta = (2\rho_0\omega h^2/\mu\eta)^2$ is the Taylor number, and $Da = K_1/h^2$ is the Darcy number. ε is the dimensionless relaxation time and ζ, the dimensionless retardation time.

We neglect the higher order terms of small quantities in Equations (12) and (13), then we perform the curl operator $(\vec{\nabla}\wedge)$ twice in order to eliminate the pressure term. the linear equations are then given by:

$$\left.\begin{array}{c} a(1+\zeta\frac{\partial}{\partial t})^2(\gamma{\nabla_h}^2 u + a\frac{\partial^2 u}{\partial z^2}) - R_D a(1+\zeta\frac{\partial}{\partial t})(1+\varepsilon\frac{\partial}{\partial t})\gamma{\nabla_h}^2\theta + \\ +(1+\varepsilon\frac{\partial}{\partial t})^2(T_D^{1/2}-c)^2\frac{\partial^2 u}{\partial z^2} = 0 \end{array}\right\}, \tag{14}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)\theta = u, \tag{15}$$

were:

$$\left.\begin{array}{c} a = cos^2\varphi + K^* sin^2\varphi, \\ c = (K^*-1)sin\varphi cos\varphi, \\ K^* = K_1/K_2, \gamma = K_1/K_3. \end{array}\right\} \tag{16}$$

and $\nabla_h{}^2 = \partial^2/\partial x^2 + \partial^2/\partial z^2$ is the horizontal Laplacian operator, $R_D = RaDa$ is Rayleigh-Darcy number and $T_D = TaDa^2$ is Taylor-Darcy number.

2.1. Rayleigh - Darcy number for anisotropic porous layer

Following the normal mode analyses, we assume that the perturbation quantities have *x, y, z* and *t* dependence of the form:

$$\left.\begin{aligned}\theta(x,y,z,t) &= \hat{\theta}(z)e^{i(lx+my)+\delta t},\\ u(x,y,z,t) &= \hat{u}(z)e^{i(lx+my)+\delta t}.\end{aligned}\right\} \tag{17}$$

where $i^2 = -1$, l and m are the horizontal wave numbers and $k = (l^2 + m^2)^{1/2}$ is the total wave number in the x-z plane. δ is the temporal growth rate. Generally, δ is a complex number and can be written as $\delta = \delta_r + \delta_i$.

Substituting Equation (17) into Equations (14) and (15), we obtain:

$$a(1+\zeta\delta)^2(aD^2 - \gamma k^2)\hat{u} + R_D ak^2(1+\zeta\delta)(1+\varepsilon\delta)\hat{\theta} + (1+\varepsilon\delta)^2(T_D^{1/2} - c)^2 D^2\hat{u} = 0, \tag{18}$$

$$(D^2 - k^2 - \delta)\hat{\theta} = -\hat{u}. \tag{19}$$

where D denotes a differential operator which is defined as $D = d/dy$. The proper boundary conditions for Equations (18) and (19) are given as:

$$\hat{u} = \hat{\theta} = D^2\hat{\theta} = 0 \quad at \quad y = 0, 1. \tag{20}$$

which indicates that the solution of $\hat{\theta}$ is of the form $\hat{\theta} = A_n sin(n\pi y)$. Eliminating $\hat{u}$ from Equations (18) and (19) and substituting this solution, one can obtain

$$R_D = \frac{n^2\pi^2 + k^2 + \delta}{k^2}\left[\frac{(1+\zeta\delta)(an^2\pi^2 + \gamma k^2)}{1+\varepsilon\delta} + \frac{\pi^2(1+\varepsilon\delta)(T_D^{1/2} - c)^2}{1+\zeta\delta}\right]. \tag{21}$$

The critical mode which minimises the Rayleigh Darcy number corresponds to $n = 1$, that is $\hat{\theta} = A_1 sin(\pi y)$ and we obtain the Rayleigh Darcy number for anisotropic porous layer:

$$R_D = \frac{\pi^2 + k^2 + \delta}{k^2}\left[\frac{(1+\zeta\delta)(a\pi^2 + \gamma k^2)}{1+\varepsilon\delta} + \frac{\pi^2(1+\varepsilon\delta)(T_D^{1/2} - c)^2}{1+\zeta\delta}\right]. \tag{22}$$

Where the parameters a and c take into account the anisotropy parameters K^* and φ (i.e. $a = cos^2\varphi + K^* sin^2\varphi, c = (K^* - 1)sin\varphi cos\varphi$ and $K^* = K_1/K_2$, $\gamma = K_1/K_3$).

When the principal axes of the porous matrix (K_1, K_2, K_3) coincides with the axial and transverse coordinates (Ox, Oy, Oz), we have $\varphi = 0^\diamond$ and $a = 1, c = 0$. Equation (22) becomes:

$$R_D = \frac{\pi^2 + k^2 + \delta}{k^2}\left[\frac{(1+\zeta\delta)(\pi^2 + \gamma k^2)}{1+\varepsilon\delta} + \frac{\pi^2 T_D(1+\varepsilon\delta)}{1+\zeta\delta}\right]. \tag{23}$$

2.2. Rayleigh-Darcy number for isotropic porous layer

When the permeability is the same in all directions (i.e. for an isotropic porous layer), we have: $K_1 = K_2 = K_3$. Which implies that $K^* = \gamma = 1$ and $a = 1, c = 0$.

The above result (Equation (22)), in the case $a = 1$, $c = 0$ and $\gamma = 1$, reduces to

$$R_D = \frac{\pi^2 + k^2 + \delta}{k^2}\left[\frac{(1+\zeta\delta)(\pi^2 + k^2)}{1+\varepsilon\delta} + \frac{\pi^2 T_D(1+\varepsilon\delta)}{1+\zeta\delta}\right]. \tag{24}$$

which corresponds to the Rayleigh Darcy number for an isotropic porous layer

2.3. Stationary convection for anisotropic porous layer($\delta = 0$)

In the case of stationary convection, R_D can be obtained from Equation (22) by letting $\delta = 0$ as follows:

$$R_{D,S} = \frac{\pi^2 + k^2}{k^2}[(a\pi^2 + \gamma k^2) + \pi^2(T_D^{1/2} - c)^2]. \tag{25}$$

2.4. Stationary convection for isotropic porous layer($\delta = 0$)

When the permeability is the same in all directions (i.e. for an isotropic porous layer), we have: $K_1 = K_2 = K_3$. Furthermore, it is easily found that, Equation (25) yields

$$R_{D,S} = \frac{\pi^2 + k^2}{k^2}[(\pi^2 + k^2) + \pi^2 T_D]. \tag{26}$$

which is reported in the past by Kang, Fu, and Tan (2011).

2.5. Critical Rayleigh-Darcy number and wave number for anisotropic porous layer($\delta = 0$)

2.5.1. Critical wave number

The critical wave number for stationary convection can be easily obtained by setting $\partial R_{D,S}/\partial k = 0$, which gives

$$k_{c,S} = \pi\left[\frac{a + (T_D^{1/2} - c)^2}{\gamma}\right]^{1/4}. \tag{27}$$

2.5.2. Critical Rayleigh-Darcy number

Substituting Equation (27) into Equation (26), we have

$$R_{D,S,c} = \gamma\pi^2\left[1 + \left(\frac{a + (T_D^{1/2} - c)^2}{\gamma}\right)^{1/2}\right]^2. \tag{28}$$

2.6. Critical Rayleigh-Darcy number and wave number for isotropic porous layer($\delta = 0$)

The above result, in the limit $a = 1$, $c = 0$ and $\gamma = 1$(i.e. isotropic porous layer), reduces to

$$k_{c,S} = \pi; R_{D,S,c} = 4\pi^2. \tag{29}$$

which are the corresponding results for viscoelastic fluids in a non-rotating porous medium, and identical to those obtained by Kim, Lee, and Chung (2003), Kang, Fu, and Tan (2011).

2.7. Overstability convection for anisotropic porous layer($\delta = i\omega$)

For the case of overstable convection the temporal growth rate δ in Equation (22) can be written as $\delta = i\omega$. Substituting $\delta = i\omega$ into Equation (22) and separating it into real and imaginary parts, then letting both of the two parts be equal to zero leads to the following equations:

$$R_{D,O} = \left.\begin{array}{l}(1/k^2)[(a\pi^2 + \gamma k^2)[(\pi^2 + k^2)(1 + \varepsilon\zeta\omega^2) + (\varepsilon - \zeta)\omega^2]/(1 + \varepsilon^2\omega^2) \\ + \pi^2(T_D{}^{1/2} - c)^2[(\pi^2 + k^2)(1 + \varepsilon\zeta\omega^2) - (\varepsilon - \zeta)\omega^2]/(1 + \varepsilon^2\omega^2)]\end{array}\right\} \tag{30}$$

where:

$$\left.\begin{array}{c}\omega^2 = (-a_4 + \sqrt{a_4 - a_5 a_6})/2a_5, \\ a_1 = (a\pi^2 + \gamma k^2), \\ a_2 = (\pi^2 + k^2)(\varepsilon - \zeta), \\ a_3 = \pi^2(T_D^{1/2} - c)^2, \\ a_4 = (a_1 + a_3)\varepsilon\zeta + a_1(1 - a_2)\zeta^2 + a_3(1 + a_2)\varepsilon^2, \\ a_5 = \varepsilon\zeta(a_1\zeta^2 + a_3\varepsilon^2), \\ a_6 = a_1(1 - a_2) + a_3(1 + a_2).\end{array}\right\} \tag{31}$$

3. Result and discussion

Figure 2(a-e), show the effect of retardation time, ζ, of permeability ratio, K^*, and of total wave number, k, on Rayleigh-Darcy number, $R_{D,O}$ for overstable convection, when $\varphi = 0°$, $\gamma = 1$, $T_D = 0.8$ and $\varepsilon = 0.6$, using Equation (30). For $K^* > 1 (K^* = 1, K^* = 2 and K^* = 5)$, Figure 2(a-c) indicate that the Rayleigh-Darcy number, $R_{D,O}$ for a given value of k, increases significantly when the retardation time increases. It can be clearly seen that the retardation time has a stabilising effect on the viscoelastic fluid saturated in the rotating anisotropic porous layer, as demonstrated by Kang, Fu, and Tan (2011) for the isotropic case for which $K^* = 1.0$. For $K^* < 1$, Figure 2(d,e) show that the retardation time has not only a stabilising effect on the viscoelastic fluid saturated in the rotating anisotropic porous layer. For example, when $K^* = 0.1$ (Figure 2(d)) and $K^* = 0.01$ (Figure 2 (e)), the Rayleigh–Darcy number, $R_{D,O}$, increases with the decrease of the retardation time when the total wave number, k, is inferior approximately to 2.8 for Figure 2 (d) (to 2.9 for Figure 2(e)) and increase with an increase of the retardation time when $k > 2.8$ for Figure 2(d) (when $k > 2.9$ for Figure 2(e)). We distinguish two zones of which the first corresponds to the effect of destabilisation of the retardation time, ζ, and the second to the effect of stabilisation. The results reveal that the permeability ratio, K^*, has a strong influence on the convection motion when the main axis of anisotropy are oriented in the same direction with the axis of coordinates (i.e. $\varphi = 0°$).

In Figure 3(a), the Rayleigh-Darcy number, $R_{D,O}$ is plotted as functions of the total wave number, k, when $\varphi = 0°$, $\gamma = 1$, $T_D = 1$ and $\varepsilon = 0.1$, for various values of $K^* = K_1/K_2$ and for a given value of the retardation time, ζ. The results clearly indicate that the anisotropy ratio K^*, has a lot of impact on the Rayleigh-Darcy number for overstable convection. We note here that the influence of the anisotropy ratio K^*, increases with an increase in the retardation time, ζ and the Rayleigh-Darcy number, for overstable convection, increases with an increase in the anisotropy ratio K^*. Therefor, the anisotropy ratio K^* has a stabilising effect on the viscoelastic fluid saturated in the rotating porous layer. This is expected, because, by noticing that $R_D = R_a Da$ where Da, depends upon of the value of K_1, an increase in Da (i.e. K_1) implies an increase in $R_{D,O}$. Figure 3(a) indicates also, that the Rayleigh-Darcy number for overstable convection, is promoted with respect to that of an isotropic porous medium corresponding to $K^* = 1.0$, when the permeability ratio K^* is made higher than 1 (i.e. $K^* = 2.0$), and is minimal for a value of K^* smaller than unity (i.e. $K^* = 0.1$), when $\varphi = 0°$.

The effects of varying K^* on the Rayleigh-Darcy number for overstable convection are illustrated in

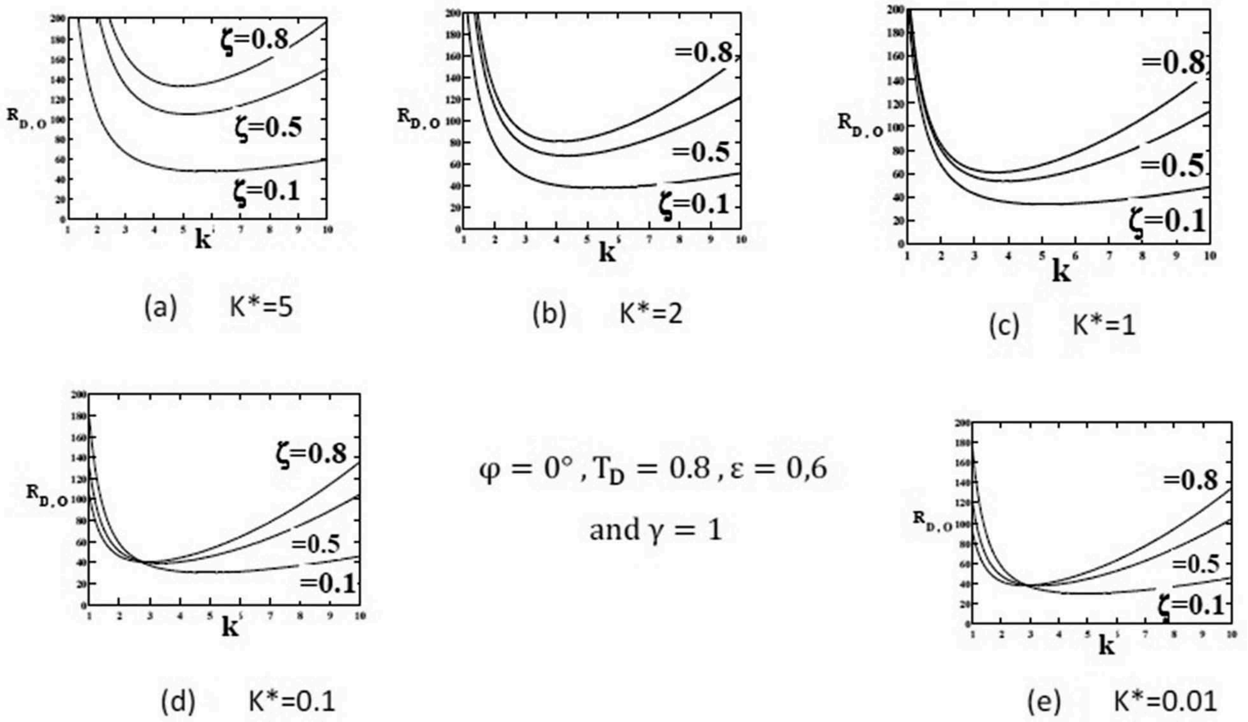

Figure 2. Effect of retardation time, ζ, of permeability ratio, K^*, and of total wave number, k, on Rayleigh-Darcy number, $R_{D,O}$.

Figure 3(b) when $\varphi = 0°$, $\gamma = 1$, $T_D = 1$ and $\zeta = 0.1$. It is noticed that the Rayleigh-Darcy number has the same behaviour as described previously in Figure 3(a), revealing that the effect of varying ε on Rayleigh-Darcy number, depends strongly on the anisotropic parameters of the porous matrix. Here, the effect of anisotropy increases with the decrease of the relaxation time, ε.

In Figure 3(c), the Rayleigh-Darcy number for overstable convection is plotted as functions of the total wave number, k, when $\varphi = 0°$, $\gamma = 1$, $\varepsilon = 0.7$ and $\zeta = 0.8$. The results indicate that the influence of the anisotropy is similar to the results observed in Figure 3(a). the Rayleigh-Darcy number, for overstable convection, increases with an increase of the Taylor-Darcy number, T_D.

The effects of varying φ, the inclination of the principal axes, on the Rayleigh-Darcy number for overstable convection are presented in Figure 4, for $K^* = 2$, $\gamma = 1$, $T_D = 0.8$, $\varepsilon = 0.6$ and for a given value of ζ. The results reveal that, for a given value of $K^* > 1$ (i.e. $K^* = 2$), the Rayleigh-Darcy number for overstable convection, $R_{D,O}$, is also affected by the anisotropic properties of the porous medium and increases with an increase in the anisotropy angle φ. $R_{D,O}$ is found to be maximum when $\varphi = 90°$ and minimum when $\varphi = 0°$. Thus, $R_{D,O}$ becomes considerably small when the orientation of the principal axis with high permeability (K_1) of the anisotropic porous medium is parallel to gravity (i.e. $K^* = K_1/K_2 = 2$ and $\varphi = 0°$). Naturally, for $K^* > 1$ the results (not presented here) indicate that, $R_{D,O}$ is promoted when the orientation of the principal axis with lower permeability (K_2) of the anisotropic porous medium is parallel to gravity (i.e. $K^* = K_1/K_2 = 2$ and $\varphi = 90°$). We note also that the influence of the anisotropy ratio K^*, decreases with an increase in the retardation time, ζ.

Figure 5(a) illustrates the effects of varying the anisotropic ratio K^* and the total wave number, k, on the Rayleigh-Darcy number of stationary convection, $R_{D,S}$ (given by Equation (25)), for $\varphi = 0°$, $\gamma = 1$ and $T_D = 1$. It is clear from Figure 5(a) that, when k is made small enough, the results indicate that the curves, for a given value of K^*, increase significantly and tend towards constant values that depend on K^* and, naturally, on φ and the Rayleigh-Darcy number of stationary convection becomes more and more affected by the anisotropic ratio K^*. Like this, $R_{D,S}$ is increased for $K^* > 1$ and reduced for $K^* < 1$. For the reason explained earlier, an increase in Da (i.e. K_1) implies an increase in $R_{D,S}$. It is seen also that the Rayleigh-Darcy number of stationary convection drops progressively with a decrease in K^* and becomes less and less affected by the anisotropic ratio K^* when the total wave number, k, is made higher.

In Figure 5(b), the Rayleigh-Darcy number of stationary convection, $R_{D,S}$, given by Equation (25), is plotted as a function of the total wave number, k, for $K^* = 2$, $\gamma = 1$ and $T_D = 0.8$ and various values of φ. For the reasons explained previously, the same trend is observed for the

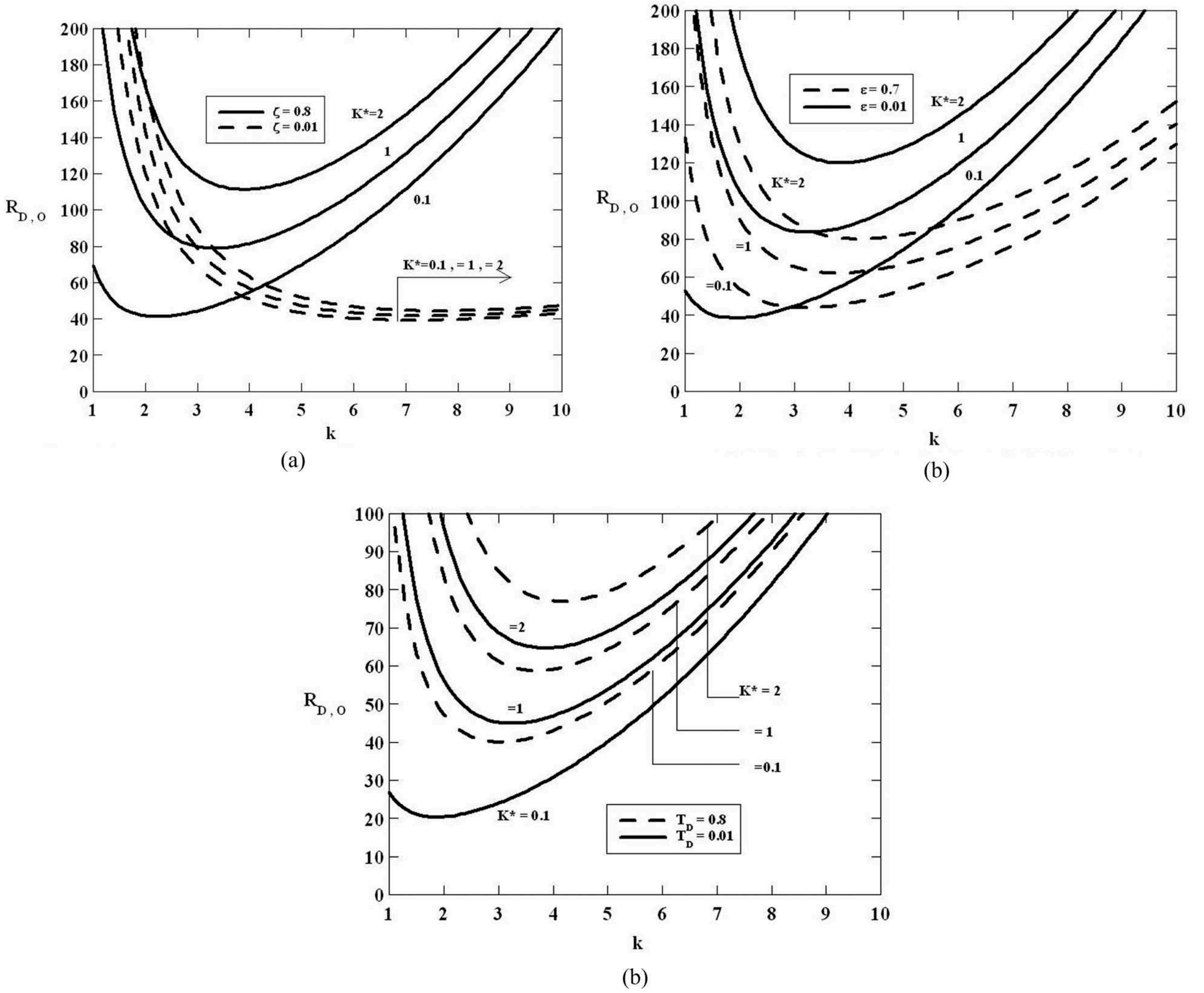

Figure 3. (a) Effect of anisotropy and retardation time on Rayleigh-Darcy number. (b) Effect of anisotropy and relaxation time on Rayleigh-Darcy number. (c) Effect of the anisotropy and of the Taylor-Darcy number on Rayleigh-Darcy number.

behaviour of the Rayleigh-Darcy number of stationary convection when the anisotropy angle, φ, varies. Moreover, contrary to the Rayleigh-Darcy number for overstable convection, the results reveal that the Rayleigh-Darcy number of stationary convection varies considerably with the angle φ and is maximum (minimum) at $\varphi = 0^\circ$ (90°) when $K^* > 1$ (i.e. $K^* = 2$) and k is made small enough.

4. Conclusion

The problem of the convective heat transfer through a viscoelastic fluid saturated rotating anisotropic porous layer heated from below has been investigated analytically. Extending the modified Darcy-Maxwell-Jeffrey model for a viscoelastic fluid in an anisotropic porous medium to include the Coriolis term, we conduct linear instability analyses of thermal convection . The effect of an anisotropy and of rotation is highlighted. The porous medium is homogeneous and assumed to be anisotropic in permeability with principal axes inclined with respect to the gravity force. The following conclusions are drawn:

(1) When the permeability ratio K^* is made higher than unity ($K^* > 1$) the Rayleigh – Darcy number of overstable convection, $R_{D,O}$, for a given value of k, increases significantly when the retardation time increases. Thus, the retardation time has a stabilising effect on the viscoelastic fluid saturated in the rotating anisotropic porous layer.
(2) When the permeability ratio K^* is made too small than unity ($K^* < 1$) the Rayleigh-Darcy number, $R_{D,O}$, increase with the decrease of the retardation time when the total wave number, k, is inferior approximately to a certain critical value of the wave number, k_{cc} which depends on K^* ($k_{cc} = 2, 8$ for $K^* = 0.1$ and $k_{cc} = 2, 9$ for $K^* = 0.01$) and increase with an increase of the retardation time when $k > k_{cc}$.
(3) The influence of the anisotropy ratio K^*, increases with an increase in the retardation time, ζ and the Rayleigh-Darcy number, for

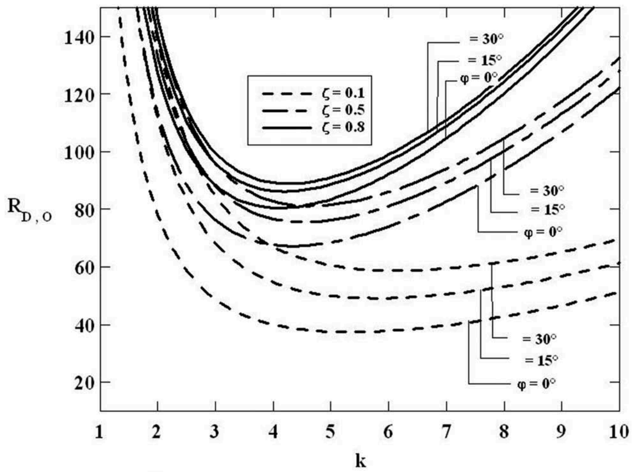

Figure 4. Effect of the orientation angle on Rayleigh-Darcy number of overstable convection.

overstable convection, increases with an increase in the anisotropy ratio K^*. The Rayleigh-Darcy number for overstable convection, is promoted with respect to that of an isotropic porous medium corresponding to $K^* = 1.0$, when the permeability ratio K^* is made higher than 1 (i.e. $K^* = 2.0$), and is minimal for a value of K^* smaller than unity (i.e. $K^* = 0.1$), when $\varphi = 0°$.

(4) The effect, of anisotropy on the Rayleigh-Darcy number of overstable convection, $R_{D,O}$, increases with the decrease of the relaxation time ε and with an increase of the Taylor-Darcy number, T_D.

(5) For a given value of $K^* > 1$ (i.e. $K^* = 2$), the Rayleigh-Darcy number for overstable convection, $R_{D,O}$, is affected by the anisotropic properties of the porous medium and increases with an increase in the anisotropy angle φ. $R_{D,O}$ is found to be maximum when $\varphi = 90°$ and minimum when $\varphi = 0°$.

(6) When k is made small enough, the Rayleigh-Darcy number of stationary convection becomes more and more affected by the anisotropic ratio K^* and the anisotropy angle φ. Like this, $R_{D,S}$ is increased for $K^* > 1$ and reduced for $K^* < 1$, and is maximum (minimum) at $\varphi = 0°$ (90°) when $K^* > 1$ (i.e. $K^* = 2$).

List of symbols

a, c	Constant, Equation (16)
$a_{i(i=1,2,3,4,5,6)}$	Constants, Equation (30)
D	Differential operator, (d/dy)
Da	Darcy number, K_1/h^2
$\vec{g}$	Gravitational acceleration
h	Vertical height (m)
$\vec{j}$	Unit vector in the y-direction
k	Thermal conductivity $(W/K.m)$
$\bar{\bar{K}}$	Flow permeability tensor, Equation (4)
K_1, K_2, K_3	Flow permeability along the principal axes
K^*, γ	Anisotropic permeability ratio, K_1/K_2, K_1/K_3, Equation (16)

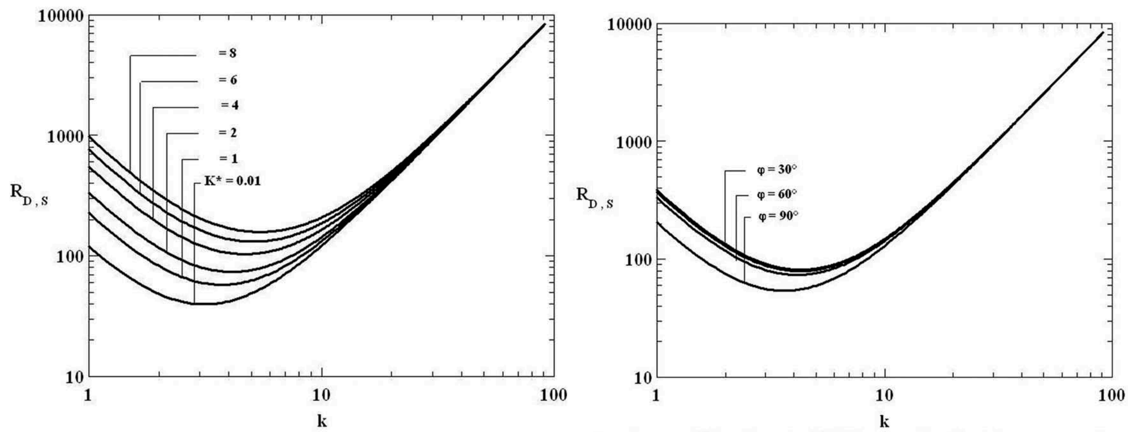

Figure 5. (a) Effect of permeability Rayleigh-Darcy number of a stationary convection. (b) Effects of the orientation angle on Rayleigh-Darcy number of a stationary convection.

P	Pressure
R_a	Rayleigh number $(\rho_o g\beta\Delta Th^3/\mu\alpha)^2$
R_D	Rayleigh-Darcy number $(RaDa)$
T	Temperature
ΔT	Temperature scale
T_o	Reference temperature
T_a	Taylor number $(2\rho_o\varpi h^2/\mu\eta)^2$
T_D	Taylor-Darcy number T_aDa^2
$\vec{V}$	Seepage velocity
u, w	Dimensionless velocity components in x, y directions
x, y, z	Dimensionless Cartesian coordinates

Greek symbols

α	Thermal diffusivity, $k/(\rho C_p)_f$
β	Thermal expansion coefficient of the fluid
δ	Complex number
ε	Dimensionless relaxation time
ζ	Dimensionless retardation time
η	Porosity
θ	Dimensionless temperature profile
φ	Inclination of the principal axis
μ	Dynamic viscosity of the fluid
ϖ	Angular velocity
ρ	Density of the fluid
σ	$= ((\rho C_p)_f\eta + (1-\eta)(\rho C_p)_s)/(\rho C_p)_f$
$(\rho c)_f$	Heat capacity of the fluid

Subscripts

ʻ	Infinitesimal perturbations
b	Basic state
c	Critical conditions
s	Stationary mode

Disclosure statement

No potential conflict of interest was reported by the authors.

References

Bertola, V., and E. Cafaro. 2006. "Thermal Instability of Viscoelastic Fluids in Horizontal Porous Layers as Initial Value Problem." *International Journal of Heat and Mass Transfer* 49: 4003–4012. doi:10.1016/j.ijheatmasstransfer.2006.04.006.

Bhatia, P. K., and J. M. Steiner. 1972. "Convective Instability in a Rotating Viscoelastic Fluid Layer." *Zeitschrift für Angewandte Mathematik und Mechanik* 52: 321–327. doi:10.1002/(ISSN)1521-4001.

Chandrasekhar, S. 1953. "The Instability of a Layer of Fluid Heated below and Subject to Coriolis Forces." *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 217: 306–327. doi:10.1098/rspa.1953.0065.

Kang, J., C. Fu, and W. Tan. 2011. "Thermal Convective Instability of Viscoelastic Fluids in a Rotating Porous Layer Heated from Below." *Journal of Non-Newtonian Fluid Mechanics* 166: 93–101. doi:10.1016/j.jnnfm.2010.10.008.

Kim, M. C., S. B. Lee, and B. J. Chung. 2003. "Thermal Instability of Viscoelastic Fluids in Porous Media." *International Journal of Heat and Mass Transfer* 46: 5065–5072. doi:10.1016/S0017-9310(03)00363-6.

Kumar, P., and H. Mohan. 2012. "Thermal Instability of a Heterogeneous Oldroydian Viscoelastic Fluid Heated from below in Porous Medium." *Journal of Theoretical and Applied Mechanics* 50 (4): 943–951.

Nield, D. A., and A. Bejan. 1992. *Convection in Porous Media*. Springer-Verlag.

Niu, J., C. J. Fu, and W. C. Tan. 2010. "Thermal Convection of a Viscoelastic Fluid in an Open- Top Porous Layer Heated from Below." *Journal of Non-Newtonian Fluid Mechanics* 165: 203–211. doi:10.1016/j.jnnfm.2009.12.002.

Patil, P. R., C. P. Parvathy, and K. S. Venkatakrishnan. 1989. "Theremohaline Instability in a Rotating Anisotropic Porous Medium." *Applied Scientific Research* 46: 7388. doi:10.1007/BF00420003.

Rajib, B., and G. C. Layek. 2012. "The Onset of Thermo-Convection in a Horizontal Viscoelastic Fluid Layer Heated Underneath." *Thermal Energy and Power Engineering* 1 (1): 1–9.

Ramesh, C., and K. Arvind. 2013. "Thermal Instability of Rotating Maxwell Viscoelastic Fluid with Variable Gravity in Porous Medium." *International Journal of Advances in Applied Mathematics and Mechanics* 1 (2): 30–38.

Saini, S., S. K. Shakya, M. P. Singh, and K. Sandeep. 2011. "Stability of Viscoelastic Fluids in Rotating Fluids through Porous Medium Heated from Below." *International Journal of Mathematical Archive* 2 (9): 1612–1618.

Sokolov, M., and R. I. Tanner. 1972. "Convective Stability of a General Viscoelastic Fluid Heated from Below." *Physical Fluids* 15: 534. doi:10.1063/1.1693945.

Sumathi, K., and S. Aiswarya. 2017. "Linear Stability of Rotating Viscoelastic Fluid Saturated Porous Layer." *International Journal of Computational and Applied Mathematics* 12 (1): 129–139.

Tan, W. C., and T. Masuoka. 2007. "Stability Analysis of a Maxwell Fluid in a Porous Medium Heated from Below." *Physics Letters A* 360: 454–460. doi:10.1016/j.physleta.2006.08.054.

Vest, C. M., and V. S. Arpaci. 1969. "Overstability of a Viscoelastic Fluid Layer Heated from Below." *Journal of Fluid Mechanisms* 36: 613–623. doi:10.1017/S0022112069001881.

Yoon, D. Y., M. C. Kim, and C. K. Choi. 2003. "Oscillatory Convection in a Horizontal Porous Layer Saturated with a Viscoelastic Fluid." *Korean Journal of Chemical Engineering* 20 (1): 27–31. doi:10.1007/BF02697180.

Yovogan, J., G. Degan, and L. Fagbemi. 2018. "Effect of Constant Magnetic Field on Convective Heat Transfer through Anisotropic River Beds." *Journal of Crystallization Process and Technology* 8: 57–71. doi:10.4236/jcpt.2018.82004.