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# INTEGRAL METHOD FOR ANALYZING NATURAL CONVECTION OF NON-NEWTONIAN FLUID ALONG A VERTICAL HEATED PLATE IN ANISOTROPIC POROUS MEDIUM

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## ABSTRACT

An analytical study of natural convection boundary-layer flow along a vertical plate embedded in an anisotropic porous medium saturated by a non-Newtonian fluid has been conducted. The principal axis of permeability anisotropy was oriented in oblique direction to the gravity vector. A power-law variation of wall temperature was prescribed at the vertical plate, and in the Darcy-Rayleigh number limit ($Ra_x > 100$), a boundary-layer solution based on the integral relations approach was assumed. Scale analysis was applied to determine the order of magnitudes involved in the boundary-layer regime. Analytical expressions were obtained for the boundary-layer thickness and the mean Nusselt number in terms of the governing parameters of the problem. When the power-law exponent was made smaller, the convective flow along the plate became stronger. When the porous matrix was oriented such that the principal axis of higher permeability was parallel to the gravity, the convective heat transfer along the plate attained maximum value.

## 1. INTRODUCTION

Buoyancy-induced flows from vertical plate embedded in fluid-saturated porous media are of technological importance in oil recovery, geothermal energy extraction, food and materials processing, dispersion of chemical contaminants, migration of moisture in insulation and grain storage, storage of radioactive materials, and ground water pollution, etc.

The problem of natural convection along a vertical heated plate with wall temperature varying as an arbitrary function of the distance from the origin has been considered [1]. Several investigations have been undertaken to incorporate boundary and inertia effects of high permeability media (excluded in Darcy's model) by assuming the fluids saturating porous media to be Newtonian. In many engineering applications, fluids saturating the porous matrices are not necessarily Newtonian.

The problem of boundary layer free convection along an isothermal vertical plate in a porous medium saturated by power-law index fluid (non-Newtonian) has been studied numerically [2]. Double diffusion from a vertical surface embedded in a porous medium saturated by a non-Newtonian fluid has also been investigated [3]. The variation of wall temperature, intensity of heat and flux of species at the wall depended strongly on the power law index [2, 3]. The porous media had been assumed isotropic, but, in several applications, the porous materials are anisotropic.

Natural convection in anisotropic porous media has not been extensively studied, except within enclosures heated from the side where one of the principal axes of anisotropy of permeability was aligned with gravity [4, 5] or all the principal axes were inclined to gravity [6, 7, 8]. In all the studies, anisotropy modified the convective heat transfer. The effects of anisotropy on boundary-layer free convection on an impermeable vertical plate had been investigated using the method of integral relations. The cases considered were coincident of the principal axes with coordinate axes [9], and arbitrary orientation of the principal axes [10]. When permeability in direction normal to the plate was found to be greater than permeability along the plate, there was increase in temperature field.

The present paper describes an integral method for obtaining approximate solution for natural convection from a vertical plate embedded in an anisotropic porous medium saturated by non-Newtonian fluid. A power law variation of wall temperature was specified on the plate, and the permeability with principal axes was oriented in a direction oblique to the gravity vector as initial conditions.

Combining modified Darcy power-law model [11] and generalized Darcy law [7], the characteristics of the saturating flow through the porous matrix was used to describe the non-Newtonian fluid behaviour. In large Rayleigh number limit, boundary layer equations were solved analytically upon introducing a scale analysis to determine the order of magnitudes involved in the boundary-layer regime.

## 2. MATHEMATICAL FORMULATION

The physical domain considered was steady heating of vertical impermeable plate embedded in a saturated non-Newtonian fluid with the porous medium characterised by anisotropic permeability of ratio $K^* = K_1/K_2$ and orientation angle $\theta$ between horizontal direction and principal axis with permeability $K_2$.

The coordinate system and flow model are illustrated in Fig.1. The x-coordinate was measured along the plate with y-coordinate normal. The gravitational acceleration g was opposite to x-direction, and the fluid and porous medium were assumed to be in thermodynamic equilibrium everywhere. The thermophysical properties of the fluid were assumed constant, except for density (buoyancy term) in the momentum equation (i.e. Boussinesq approximation).

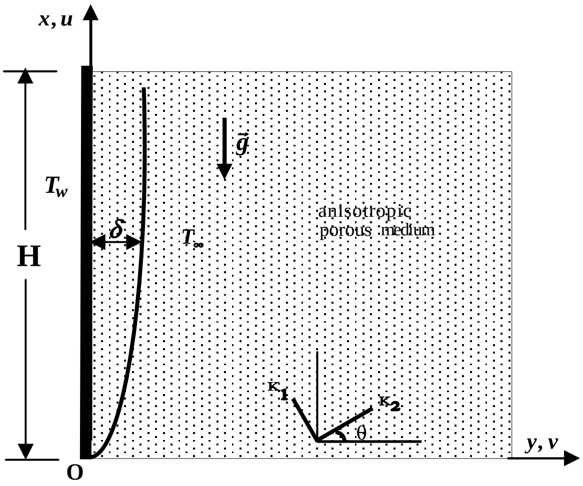

Fig. 1. Coordinate system and flow model (T is temperature, H is height of plate and δ is thickness of vertical boundary layer).

The model of laminar flow of a non-Newtonian power-law fluid through the porous medium described by generalized Darcy law was written as [7, 11],

$$\vec{V} = -\frac{\overline{\overline{K}}}{\mu_a}\nabla p \qquad (1)$$

where $\mu_a = \varepsilon \left(u^2 + v^2\right)^{(n-1)/2}$, and $\varepsilon = 2\mu/8^{(n+1)/2}(\sqrt{K_1 K_2}\,\gamma)^{(n-1)/2}(1+3n)^n$, where $\vec{V}$ is superficial velocity, $\gamma$ is porosity of medium, $\mu_a$ is apparent viscosity, μ is consistency index, n is power-law index, and u and v represent velocity components in x and y directions. The rheological parameters $\mu$ and n were assumed to be temperature independent.

The equations describing conservation of mass, momentum, energy and the state for the physical domain were respectively [12],

$$\nabla.\vec{V} = 0 \qquad (2)$$

$$\vec{V} = -\frac{\overline{\overline{K}}}{\mu_a}(\nabla p + \rho\vec{g}) \qquad (3)$$

$$\nabla.\left(\vec{V}T - \alpha\,\nabla T\right) = 0 \qquad (4)$$

$$\rho = \rho_\infty\left[1 - \beta\left(T - T_\infty\right)\right] \qquad (5)$$

where T is local equilibrium temperature of fluid and porous matrix, $\vec{g}$ is gravitational acceleration, p is pressure, α is thermal diffusivity $\alpha = k/(\rho c_p)_f$, k is thermal conductivity of fluid/porous matrix combination, $(\rho c_p)_f$ is heat capacity of the fluid, β is coefficient of thermal expansion of the fluid and ρ is density.

The boundary conditions associated with the governing eqns. (2 – 5) were

$$\text{for } y = 0\text{:} \quad v = 0,\ T = T_w = T_\infty + Ax^\lambda \qquad (6)$$

$$\text{and } y \to \infty\text{:} \quad u = 0,\ T = T_\infty \qquad (7)$$

where the prescribed wall temperature $T_w$ was a power function of distance x from the edge of the plate, A and $\lambda$ are positive constants, and $T_\infty$ is temperature of the porous medium far away from the wall. If constant heat flux was prescribed at the plate, then $\lambda = n/(2n + 1)$ and $\lambda = 0$ for the isothermal plate [3].

The symmetrical second-order permeability tensor $\overline{\overline{K}}$ was defined as,

$$\overline{\overline{K}} = \begin{bmatrix} K_1\cos^2\theta + K_2\sin^2\theta & (K_1 - K_2)\cos\theta\sin\theta \\ (K_1 - K_2)\cos\theta\sin\theta & K_2\cos^2\theta + K_1\sin^2\theta \end{bmatrix} \qquad (8)$$

Eliminating the pressure term by the curl of eqn. (3) and substituting eqn. (2) a single momentum equation obtained was,

$$a\frac{\partial u}{\partial y} + c\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) - b\frac{\partial v}{\partial x} = \frac{1}{\mu_a}\left\{\frac{\partial \mu_a}{\partial x}(-cu + bv) + \frac{\partial \mu_a}{\partial y}(-au + cv) + K_1\rho_\infty\beta g\frac{\partial T}{\partial y}\right\} \qquad (9)$$

Where $a = \cos^2\theta + K^*\sin^2\theta$, $b = \sin^2\theta + K^*\cos^2\theta$;

and $c = \frac{1}{2}(1 - K^{*}\ )\sin 2\theta$ (10)

## 3. SCALE ANALYSIS OF MAGNITUDE OF VARIABLES

The boundary-layer regime for which fluid motion was restricted to a thin layer $\delta$ along vertical plate, momentum eqn. (9), and the boundary-layer hypothesis could only be used when the conditions;

$$a\frac{\partial u}{\partial y} \gg c\frac{\partial v}{\partial y}, \qquad a\frac{\partial u}{\partial y} >> c\frac{\partial u}{\partial x}, \qquad a\frac{\partial u}{\partial y} >> b\frac{\partial v}{\partial x},$$

$K_1\rho_\infty\beta g\frac{\partial T}{\partial y} >> \frac{\partial\mu_a}{\partial x}\left(-c\,u + b\,v\right)$, and

$K_1\rho_\infty\beta g\frac{\partial T}{\partial y} >> \frac{\partial\mu_a}{\partial y}\left(-a\,u + c\,v\right)$ were satisfied. For boundary-layer approximations at large Rayleigh number, governing eqns. (2 - 4) become conservation equations,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{11}$$

$$\frac{\partial}{\partial y}\left(u^{n}\right) = \frac{n}{a}\frac{K_1\rho_\infty g\beta}{\varepsilon}\frac{\partial T}{\partial y} \tag{12}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{13}$$

By eqn. (10), and assuming H and δ are the x and y scales in the boundary-layer ($\delta \ll H$), the conservation eqns. (11 - 13) required the functional balances,

$$\frac{u}{H} \sim \frac{v}{\delta}, \qquad a\frac{u}{\delta} \sim \frac{1}{\varepsilon u^{n-1}}K_1\rho_\infty g\beta\frac{\Delta T}{\delta},$$

$$u\frac{\Delta T}{H} \text{ and } v\frac{\Delta T}{\delta} \sim \alpha\frac{\Delta T}{\delta}$$

where $\Delta T\left(= T_W - T_\infty\right)$ is characteristic scale of temperature, and temperature drop across the boundary layer was of order one.

Solving the balance for $\delta$, $u$ and $v$,

$$\delta \sim H Ra_H^{-1/(2n)} a^{1/(2n)} \tag{14}$$

$$u \sim \frac{\alpha}{H} Ra_H^{1/n} a^{-1/n} \tag{15}$$

$$v \sim \frac{\alpha}{H} Ra_H^{1/(2n)} a^{-1/(2n)} \tag{16}$$

## 4. DEFINITIONS & CONDITIONS

The validity of boundary layer analysis was considered. A modified Darcy Raylei-gh number, $Ra_H$, based on height of the plate was defined as, $Ra_H = K_1\rho g\beta\Delta T H^{n}/\varepsilon\alpha^{n}$ [2].

A stream function $\psi$ related to velocity components was defined as [8]

$$u = \frac{\partial\psi}{\partial y}, \qquad v = -\frac{\partial\psi}{\partial x} \tag{17}$$

such that the continuity eqn. (11) was satisfied, and scale for the stream function written as,

$$\psi \sim \alpha Ra_H^{1/(2n)} a^{-1/(2n)} \tag{18}$$

The local Nusselt number, $Nu_H$, defined as the heat transfer over pure heat conduction through the vertical plate was scaled as,

$$Nu_H = \frac{hH}{k} \sim Ra_H^{1/(2n)} a^{-1/(2n)} \tag{19}$$

where $h = q/\left(T_W - T_\infty\right)$ is local heat transfer coe-fficient, and $q = -k/\left(\partial T/\partial y\right)\Big|_{y=0}$ is local surf-ace heat flux at the heated plate.

For isotropic porous medium where K* = 1 (i.e. a = 1), scales for the stream function were reduced to double diffusion boundary layer over vertical plate embedded in porous region saturated by non-Newtonian fluid [3].

The results would be valid only when the vertical boundary-layer was thin ($\delta \ll H$), i.e. for $Ra_H \gg a$. From boundary layer conditional requirements of eqns. (14) - (16), and the results of order-of-magnitude analysis, the boundary layer hypothesis was valid only when the conditions $b \ll Ra_H^{1/n} a^{(n-1)/2n}$ and $c \ll Ra_H^{1/(2n)} a^{(2n-1)/2n}$ were satisfied.

## 5. METHOD OF SOLUTION

The governing equations were solved to obtain analytical expressions for the boundary layer thickness and mean Nusselt number in term of the governing parameters. The governing eqns. (11) - (13) and boundary conditions eqns. (6) and (7) were integrated by the Karman-Pohlhausen method to obtain the heat transfer rate through the vertical plate.

Integrating eqn. (10) with limits y = 0 to Y (region where $Y \geq \delta$ is situated in the free stream), and considering boundary conditions,

$$u^{n} = \frac{nK_1\rho_\infty g\beta}{a\varepsilon}\left(T - T_\infty\right) \tag{20}$$

From continuity eqn. (11), the energy eqn. (13) was re-written as,

$$\frac{\partial}{\partial x}(uT)+\frac{\partial}{\partial y}(vT)=\alpha\frac{\partial^2 T}{\partial y^2} \qquad (21)$$

and upon integrating over the region $\delta \times H$, where $(\partial / \partial y)\big|_{y=\delta} = 0$, yielded

$$\frac{d}{dx}\int_0^{\delta} uT\,dy=-\alpha\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad (22)$$

Substituting eqn. (20) into eqn. (22) and re-arranging,

$$-\alpha\left(\frac{\partial \Phi}{\partial y}\right)_{y=0}=\left(\frac{n}{a}\frac{K_1\rho_\infty g\beta}{\varepsilon}\right)^{1/n}\frac{d}{dx}\int_0^{\delta}\Phi^{(n+1)/n}dy \qquad (23)$$

where $\Phi = T - T_\infty$.

Introducing transformations obtained by boundary-layer scales, the transformation profile of unknown function η was

$$\Theta(\eta) = \Phi/\Phi_W \qquad (24)$$

where $\eta = y/\delta$ and $\Phi_W = T_W - T_\infty$.

After algebraic manipulations, eqn. (23) became,

$$\frac{\delta}{x}=\left\{\frac{2n}{(2n+1)\lambda+n}\frac{J}{I}n^{-1/n}\right\}^{1/2} Ra_x^{-1/(2n)}a^{1/(2n)} \qquad (25),$$

an expression of modified Darcy-Rayleigh number based on the permeability $K_1$ and distance *x* from the edge of the plate where

$$J = -\Theta'(0) \qquad (26),$$

$$I = \int_0^1 \Theta^{(n+1)/n}(\eta)\,d\eta \qquad (27),$$

and $Ra_x = K_1\rho_\infty\beta A x^{\lambda+n}/\varepsilon\alpha^n$.

Hence, eqn. (25) is in good agreement with eqn. (14) predicted by the scale analysis.

The expressions of *I* and *J* (eqns. (26, 27) depended on temperature profile $\Theta(\eta)$, therefore, *I* and *J* can be constants if the profile $\Theta(\eta)$ is determined. By the boundary thermal condition eqns. (12) and (13), the temperature profile satisfied the relations,

$$\Theta(0) = 1(a); \qquad \Theta(1) = 0(b) \qquad (28)$$

The method of solution adopted was choosing expression of the temperature profile $\Theta$, which satisfied conditional eqn. (28).

The value of $\Theta$ decreased from 1 to 0, as $\eta$ increased from 0 at different values of anisotropic parameter *a* of porous matrix and power-law index *n* of saturating fluid. From eqn. (25), the boundary layer thickness, $\eta_\delta$, was expressed as,

$$n_\delta=\left\{\frac{2n}{(2n+1)\lambda+n}\frac{J}{I}n^{-1/n}\right\}^{1/2} a^{1/(2n)} \qquad (29)$$

The local heat flux *q* along the vertical plate then became

$$q=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}=k\frac{\Phi_W}{\delta}\left[-\Theta'(0)\right]=k\frac{\Phi_W}{\delta}J \qquad (30)$$

and by combining eqns. (19), (25) and (30), local Nusselt number $Nu_x$ was rewritten as,

$$Nu_x=\left\{\frac{(2n+1)\lambda+n}{2n}IJn^{-1/n}\right\}^{1/2} Ra_x^{1/(2n)}a^{-1/(2n)} \qquad (31)$$

Equation (31) is compatible with eqn. (19) obtained by scale analysis.

## 6. RESULTS

The integral method was validated by comparing values of parameter $Nu_x/(Ra_x)^{1/2}$ shown in Table 1 with values from solution of the full governing boundary-layer equations using both consistent numerical procedure and similarity method when the porous matrix was anisotropic in permeability [10]. Setting $\lambda = 0$ in the limit of constant wall temperature when the porous matrix was saturated by Newtonian fluid (n = 1), the solution presented in the present analysis was in agreement with results reported [10].

Table 1. Values of $Nu_x/(Ra_x)^{1/2}$ by different method for $n = 1$ at constant temperature at wall ($\lambda = 0$).

| $a = K^*$ ($\theta = 90^\circ$) | $Nu_x/(Ra_x)^{1/2}$ | |
|---|---|---|
| | **Similarity solution [10]** | **Integral method** $\left(=(IJ/(2a))^{1/2}\right)$ |
| 0.1(< 1.0) | 1.404 | 1.337 |
| 1.0 (isotropic) | 0.444 | 0.423 |
| 10. (> 1.0) | 0.140 | 0.134 |

Heating the plate by constant thermal flux [3], λ depended on power-law exponent of fluid saturating the porous matrix, expressed by $\lambda = n/(2n+1)$. Therefore, in the limit of Newtonian fluids (n=1), and for λ = 1/3. Table 2 shows the result obtained by integral method as compared with similarity solution.

Table 2. Values of $Nu_x/(Ra_x)^{1/2}$ by different methods for n = 1 and constant heat flux at the wall (λ = 1/3)

| $a = K^*$ ($\theta = 90^\circ$) | $Nu_x/(Ra_x)^{1/2}$ | |
|---|---|---|
| | **Similarity solution 10]** | **Integral method** ($= (IJ/a)^{1/2}$) |
| 0.1(< 1.0) | 2.1465 | 2.0564 |
| 1.0 (isotropic) | 0.6788 | 0.6503 |
| 10. (> 1.0) | 0.2146 | 0.2056 |

Figure 2 shows the local Nusselt number parameter $\mathrm{Nu_x}/(\mathrm{Ra_x})^{1/(2n)}$, given as a function of anisotropic permeability ratio $K^*$ and inclination angle θ of principal axis of porous medium, for wall heated isothermally (λ = 0) and for different values of n, where in the limit of Newtonian fluid (n = 1) the data agreed with reported results [10].

The influence of anisotropy orientation θ on local Nusselt number, $Nu_x$, is presented in Fig. 3 where the plate was heated isothermally (λ = 0) and dilatant fluid (n = 2*)* saturated the porous matrix for values of $K^*$.

Fig. 4 shows the effect of modified Darcy-Rayleigh number $R_H$, on mean Nusselt number $\overline{\mathrm{Nu}}_{0-H}$ for K* = 10, θ = 90°, and for different power-law indexes *n*, when the plate was heated by a constant flux.

From eqn. (19), $\overline{\mathrm{Nu}}_{0-H}$ over the vertical plate of height H was deduced as,

$$\overline{\mathrm{Nu}}_{0-H} = \overline{\mathrm{h}}_{0-H}\,\frac{\mathrm{H}}{\mathrm{k}} \tag{32}$$

Where $\overline{\mathrm{h}}_{0-H} = \int_0^H \mathrm{q}\,\mathrm{dx} \Big/ \mathrm{H}\left(\overline{\mathrm{T_w} - \mathrm{T_\infty}}\right)$ (33)

and $\left(\overline{\mathrm{T_W} - \mathrm{T_\infty}}\right)$ is mean temperature difference along the vertical plate.

Combining eqns. (25), (30) and (32),

$$\overline{\mathrm{Nu}}_{0-H} = \left\{\frac{2n^{(n+1)/n}(1+\lambda)^{(2n+1)/n}}{(2n+1)\lambda + n}\,\mathrm{IJ}\right\}^{1/2} \mathrm{R_H}^{1/(2n)}\,a^{-1/(2n)} \tag{34}$$

where $\mathrm{Ra_x} = \mathrm{K_1}\rho_\infty\beta(\overline{\mathrm{T_w} - \mathrm{T_\infty}})\,\mathrm{H}^n/\varepsilon\alpha^n$ is modified Darcy-Rayleigh number based on mean temperature difference $\left(\overline{\mathrm{T_W} - \mathrm{T_\infty}}\right)$ over the vertical surface. Equation (34) is also compatible with eqn. (19).

The asymptotic behaviour of the temperature profile at neighbourhood of the vertical plate, indicated that suitable expression of the temperature distribution temperature could be written as,

$$\Theta(\eta) = 1 - \frac{17}{5}\eta + \frac{12}{5}\eta^2 \tag{35}$$

to satisfy the boundary condition eqn. (28).

## 6. DISCUSSION

Steady state natural convection along vertical plate embedded in an anisotropic porous medium where the principal axes were not

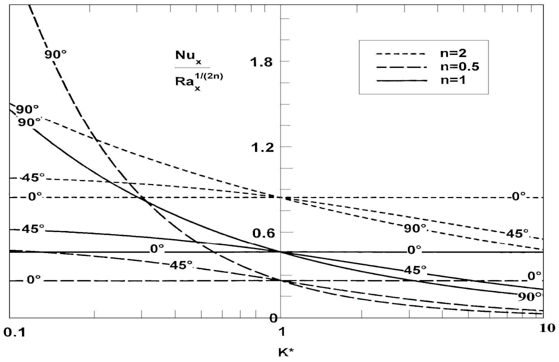

Fig. 2. Variation of local Nusselt number over the plate versus K* for values of θ and n when λ= 0.

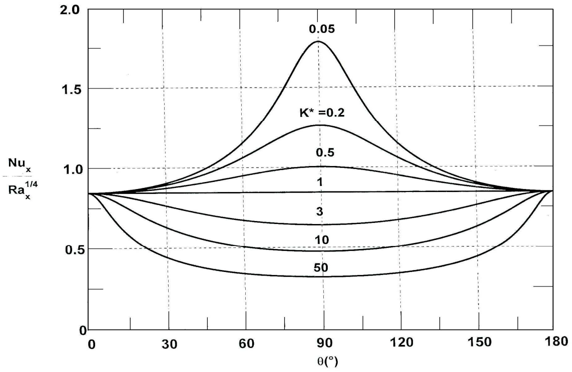

Fig. 3. Effect of anisotropic orientation angle θ on local Nusselt number for *n* = 2 and values of K*, when λ = 0.

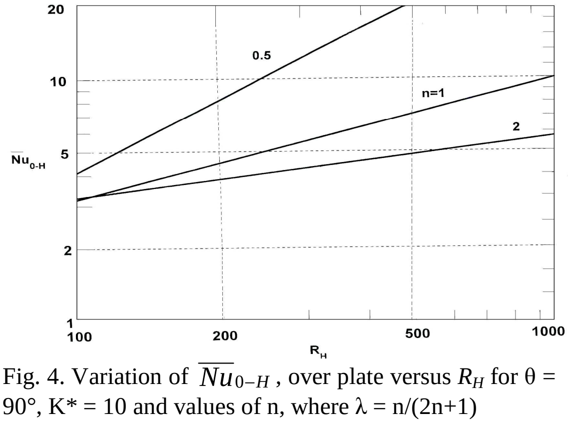

Fig. 4. Variation of $\overline{Nu}_{0-H}$ , over plate versus $R_H$ for θ = 90°, K* = 10 and values of n, where λ = n/(2n+1)

coincident with the gravity vector, and were analysed on the basis of modified Darcy power law model [11] and generalized Darcy law [7]. In the boundary-layer regime, an integral method valid for non-Newtonian fluids was developed, and the results obtained were in agreement with the order of magnitudes determined by scaling analysis. The mean Nusselt number was obtained as a function of modified Darcy-Rayleigh number, power law index of fluids and anisotropic parameters of the porous medium.

The parameter $Nu_x/(Ra_x)^{1/2n}$ depended on the heating process (temperature profile through the constants I and J), and $a = \cos^2\theta + K^* \sin^2\theta$ (anisotropic parameters). Fig. 2 illustrates the observation for the curves in dashed lines (in the limit of a pseudoplastic fluid n = 0.5), and in dotted lines (in the limit of dilatant fluid n = 2 saturating the porous matrix).

When $\theta = 0°$ ($a = 1$), Fig. 2 indicates that for each value of n, $Nu_x/(Ra_x)^{1/2n}$ is constant and independent of K*. Hence, the local heat transfer in the boundary-layer regime depended only on Rayleigh number $Ra_x$ based on permeability $K_1$ along the plate, and was not affected by magnitude of permeability $K_2$ in the direction normal.

As shown in Fig. 2, for $\theta = 90°$ ($a = K^*$) permeability $K_2$ was aligned along the plate while permeability $K_1$ was in orthogonal direction, and the resulting local Nusselt number depended strongly on permeability ratio K*. The trend follows since $Ra_x$ was based on $K_1$ being perpendicular to the plate; which is not appropriate parameter for the situation. Therefore, considering eqn. (31) and the Rayleigh number ($Ra_x/a = K_2\rho_\infty g\beta A x^{\lambda+n}/(\varepsilon\alpha^n)$) based on permeability $K_2$ along the plate, $Nu_x$ becomes independent of K*.

For $\theta = 45°$, K* becomes larger as value of $Nu_x$ decreases, since for a given Rayleigh number (i.e., a given value of $K_1$) an increase in K* corresponded to a decrease in $K_2$, resulting in a weaker convection flow and heat transfer rate [10].

In isotropic medium ($K^* = 1$, $a = 1$), $Nu_x/(Ra_x)^{1/4}$ is independent of θ. The symmetry of the results with respect to $\theta = 90°$ as shown in Fig. 3 is evidenced by the governing eqns. (9) and (13) and boundary conditions eqns. (6) and (7); i.e. if $\psi(x, y)$ and $T(x, y)$ are solutions for $Ra_H$, K* and θ, then $\psi(x, 1-y)$ is a solution for $Ra_H$, K* and $\pi - \theta$.

For the range $0 < \theta < 90°$ and $K^* < 1$, Fig. 3 indicates that $Nu_x/(Ra_x)^{1/4}$ is minimum at $\theta = 0°$, for which permeability in vertical direction is minimum, but is maximum at $\theta = 90°$. The inverse is observed for $K^* > 1$ where the convective heat transfer is maximum at $\theta = 0°$ and minimum at $\theta = 90°$. The observation can be demonstrated for the solution $Nu_x/(Ra_x)^{1/4} = 0.8436/a^{1/4}$.

Taking first derivative of $Nu_x/(Ra_x)^{1/4}$ with respect to θ and equating to zero, $(K^* - 1)\sin 2\theta = 0$, such that a maximum or a minimum occurs for $\theta = 0°$ and $\theta = 90°$.

The second derivative of $Nu_x/(Ra_x)^{1/4}$ with respect to θ equals $1.6872(1-K^*)$ when $\theta=0°$, and $1.6872\,(K^*-1)(K^*)^{-5/4}$ when $\theta=90°$.

When $K^*>1$ ($K^*<1$), $Nu_x/(Ra_x)^{1/4}$ is maximum (minimum) at $\theta = 0°$ and minimum (maximum) at $\theta = 90°$. Therefore, a maximum (minimum) convective heat transfer is reached when orientation of the principal axis with higher permeability of anisotropic porous medium is parallel (perpendicular) to the gravity, as reported on the effect of anisotropic permeability of arbitrary orientation on the convective heat transfer in a vertical porous cavity heated isothermally from the side [6, 7].

The exponent in power law variation was expressed as $\lambda = n/(2n + 1)$, and the plots in Fig. 4 are straight lines with different gradients depending on n. A higher power-law index requires large modified Darcy-Rayleigh number for the boundary-layer regime. When Rayleigh number is large or power-law index is small, the convective heat transfer becomes stronger. The deduction agrees with the findings obtained for a boundary-layer natural convection in a vertical porous layer filled with a non-Newtonian fluid [12].

## 7. CONCLUSIONS

The convective flow along a vertical plate embedded in an anisotropic porous medium saturated by non-Newtonian fluid is considerably affected by anisotropic parameters of permeability ratio K* and inclination angle θ the principal axes, and the index (n) of non-Newtonian saturating fluid.

A maximum (minimum) convective heat transfer along the vertical plate was obtained when the porous matrix was oriented such that the principal axis with higher permeability was parallel (perpendicular) to the vertical direction.

When the modified Darcy-Rayleigh number was made larger for lower power-law index (n < 1 characterizing shear-thinning fluids), the convective heat transfer through the vertical plate became stronger, a concept useful in choosing fluids for given applications.

## 8. REFERENCES

1. Seetharamu K.N. and Dutta P., Free convection in a saturated porous medium adjacent to a non-isothermal vertical impermeable wall, Effects of anisotropy on the free convection from a concentrated source in a porous medium, *Wärme-und-Stoffübertragung,* Vol. 25, 1990, pp. 9 - 15.
2. Chen H.T. and Chen C.K., Free convection flow of non-Newtonian fluids along a vertical plate embedded in a porous medium, *J. Heat Transfer,* Vol. 110, 1988, pp. 257 - 260.
3. Rastogi S.K. and Poulikakos D., Double-diffusion from a vertical surface in a porous region saturated with a non-Newtonian fluid., *Int. J. Heat Mass Transfer,* Vol. 38, 1995, pp. 935 - 946.
4. Kimura S., Masuda Y. and Kazuo H.T., Natural convection in an anisotropic porous medium heated from the side (effects of anisotropic properties of porous matrix), *Heat transfer Jpn. Res.,* Vol. 22, 1993, pp. 139 - 153.
5. Ni J. and Beckermann C., Natural convection in a vertical enclosure filled with anisotropic permeability, *J. Heat Transfer,* Vol. 113, 1993, pp. 1033 - 1037.
6. Zhang X., Convective heat transfer in a vertical porous layer with anisotropic permeability, in *Proc. 14th Canadian Congr. Appl. Mech.,* Vol. 2, 1993, pp. 579 - 580.
7. Degan G., Vasseur P. and Bilgen E., Convective heat transfer in a vertical anisotropic porous layer, *Int. J. Heat Mass Transfer,* Vol. 38, 1995, pp. 1975 - 1987.
8. Degan G. and Vasseur P., Natural convection in a vertical slot filled with an anisotropic porous medium with oblique principal axes, *Numerical Heat Transfer,* A30, 1996, pp. 397 - 412.
9. Ene H. I., Effects of anisotropy on the free convection from a vertical plate embedded in a porous medium, *Trans. Porous Media,* Vol. 6, 1991, pp. 183 - 194.
10. Vasseur P. and Degan G., Free convection along a vertical heated plate in a porous medium with anisotropic permeability, *Int. J. of Numerical Methods for Heat and Fluid Flow,* Vol. 8, 1998, pp. 43 - 63.
11. Pascal H., Rheological behaviour effect of non-Newtonian fluids on steady and unsteady flow through porous media, *Int. J. Numerical and Analytical Methods in Geomechanics,* Vol.7, 1983, pp. 207 - 224.
12. Bian W., Vasseur P. and Bilgen E., Boundary-layer analysis for natural convection in a vertical porous layer filled a non- Newtonian fluid, *Int.J. Heat and Fluid Flow,* Vol. 1, 1994, pp**.** 384 – 391.

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