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Gausson parameter dynamics in ENZ-material based waveguides using moment method

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ABSTRACT

In this article, we study the dynamics of optical soliton (Gausson) pulses in waveguides based on ENZ-materials. The theoretical approach used is based on the handling of the non-linear Schrödinger's equation using the moment method. The physical system studied is dissipative. The loss of energy in the waveguide, which can be explained by the existence of imaginary parts of the refractive index of the ENZ-material, is negligible because permittivity of ENZ–materials is low. In addition, we show that self-phase modulation results in the increase of chirp during the propagation of such Gaussons.

1. Introduction

The study of linear and non-linear effects that appear during the propagation of ultrashort pulses in waveguides often requires the solution of the associated non-linear Schrödinger's equation (NLSE) [1,13]. The study of NLSE has been carried out in a wide variety of circumstances as seen globally [1–30]. Thus solving equations, in most cases, allows to find an approximate solution which gives the behavior of the perturbed Gaussons because of dispersion, gain, cubic–quintic–septic non–linear effects and absorption. This is how the current work will deal with, namely self–phase modulation effect waveguides based on ENZ–materials. In addition to the self-phase modulation effect, we are going to focus on the variation of energy, Gausson width, shift, chirp and frequency by the aid of the moment method. The basic purpose of the work is to choose the governing equation that makes it essential for ideal Gausson

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transmission dynamics. Referring to the extraordinary properties of ENZ-materials [14,21], we will use the following equation:

$$-i\psi_z-\frac{K_2}{2}\psi_{\tau\tau}+i\frac{K_3}{6}\psi_{\tau\tau\tau}+a\mid\psi\mid^2\psi+\mathrm{i}a_1\left(\left|\psi\right|^2\psi\right)_\tau+b\mid\psi\mid^4\psi$$
$$+\mathrm{i}b_1\left(\left|\psi\right|^4\psi\right)_\tau+d\mid\psi\mid^6\psi+\mathrm{i}d_1\left(\left|\psi\right|^6\psi\right)_\tau=0. \tag{1}$$

Here, in Eq. (1), $\psi=\psi(z,\tau)$, $\psi_z=\frac{\partial\psi}{\partial z}$, $\psi_{\tau\tau}=\frac{\partial^2\psi}{\partial\tau^2}$, $K_2=\beta_2+i\alpha_2$, $K_3=\beta_3+i\alpha_3$, $a=\gamma_0+i\delta_1$, $a_1=\gamma_s+i\delta_2$, $b=\gamma+i\delta_3$, $b_1=\gamma_1+i\delta_4$, $d=\theta+i\delta_5$, $d_1=\theta_1+i\delta_6$ where β_2 and β_3 are respectively the second and the third order dispersion parameters; γ_0 and γ_s are the third parameters; γ and γ_1 are the quintic parameters; θ and θ_1 are the septic parameters; γ_0, γ, and θ are self-phase modulation (SPM) parameters; γ_s, γ_1, and θ_1 are self steepening parameters; α_2 and α_3 are gain bandwidth parameters; δ_1, δ_2, δ_3, δ_4, δ_5 and δ_6 are respectively cubic-1, cubic-2, quintic-1, quintic-2, septic-1 and septic-2 nonlinear absorption parameters.

2. Parameter evolution

The aim to study NLSE in optics is to investigate the evolution of the pulse through a material. Thus, the optical pulse is seen as a moving particle [22]. To this end, we associate optical pulse energy E, the shift T, the width σ, the frequency Ω, and the chirp C. Considering these parameters, our study will therefore focus on their evolution. The parameters of the pulse are defined from the moment method as:

$$E=\int_{-\infty}^{\infty}\mid\psi\mid^2\mathrm{dt} \tag{2}$$

$$T=\frac{1}{E}\int_{-\infty}^{\infty}t\mid\psi\mid^2\mathrm{dt} \tag{3}$$

$$\Omega=\frac{i}{2E}\int_{-\infty}^{\infty}\left(\psi^*\psi_t-\psi\psi_t^*\right)\mathrm{dt} \tag{4}$$

$$\sigma^2=\frac{1}{E}\int_{-\infty}^{\infty}(t-T)^2\mid\psi\mid^2\mathrm{dt} \tag{5}$$

$$C=\frac{i}{2E}\int_{-\infty}^{\infty}(t-T)\left(\psi^*\psi_t-\psi\psi_t^*\right)\mathrm{dt} \tag{6}$$

where $\psi=\psi(z,t)$ is a trial function which best reproduces the behavior of the solution as is appropriate in any variational method, $\psi*$ is the complex conjugate of ψ, $\psi_t=\frac{\partial\psi}{\partial t}$ is the partial derivative of ψ with respect to t and ψ is a summable square function.

2.1. Energy evolution

The variation of the energy E of the pulse is expressed as:

$$\frac{\mathrm{dE}}{\mathrm{dz}}=\int_{-\infty}^{\infty}\left(\psi^*\psi_z+\psi\psi_z^*\right)\mathrm{dt}. \tag{7}$$

Using (1) and (7) after performing calculations we have:

$$\frac{\mathrm{dE}}{\mathrm{dz}}=\alpha_2\int_{-\infty}^{\infty}\mid\psi_t\mid^2\mathrm{dt}+i\frac{\alpha_3}{6}\int_{-\infty}^{\infty}\left(\psi_t^*\psi_{tt}-\psi_t\psi_{tt}^*\right)\mathrm{dt}+2\delta_1\int_{-\infty}^{\infty}\mid\psi\mid^4\mathrm{dt}$$
$$+2\delta_3\int_{-\infty}^{\infty}\mid\psi\mid^6\mathrm{dt}+2\delta_5\int_{-\infty}^{\infty}\mid\psi\mid^8\mathrm{dt}+i\delta_2\int_{-\infty}^{\infty}\mid\psi\mid^2\left(\psi^*\psi_t-\psi\psi_t^*\right)\mathrm{dt} \tag{8}$$
$$+i\delta_4\int_{-\infty}^{\infty}\mid\psi\mid^4\left(\psi^*\psi_t-\psi\psi_t^*\right)\mathrm{dt}+i\delta_6\int_{-\infty}^{\infty}\mid\psi\mid^6\left(\psi^*\psi_t-\psi\psi_t^*\right)\mathrm{dt}.$$

2.2. Shift evolution

For the study of the evolution of the shift, we refer to (3) and we have:

$$\frac{\mathrm{dT}}{\mathrm{dz}}=\frac{1}{E}\int_{-\infty}^{\infty}t\left(\psi^*\psi_z+\psi\psi_z^*\right)\mathrm{dt}-\frac{T}{E}\frac{\mathrm{dE}}{\mathrm{dz}}. \tag{9}$$

By introducing (1) in (9) we have:

$$\frac{\mathrm{dT}}{\mathrm{dz}}=\frac{I}{E}-\frac{T}{E}\frac{\mathrm{dE}}{\mathrm{dz}} \tag{10}$$

where

$$\begin{aligned} I = \; & -\beta_2 E\Omega+\alpha_2\int_{-\infty}^{\infty} t\,|\,\psi_t|^2\mathrm{dt}+i\frac{\alpha_3}{6}\int_{-\infty}^{\infty} t\left(\psi_t^*\psi_{\mathrm{tt}}-\psi_t\psi_{\mathrm{tt}}^*\right)\mathrm{dt} \\ & +2\delta_3\int_{-\infty}^{\infty} t\,|\,\psi|^6\mathrm{dt}+2\delta_5\int_{-\infty}^{\infty} t\,|\,\psi|^8\mathrm{dt}+i\delta_2\int_{-\infty}^{\infty} t\,|\,\psi|^2\left(\psi^*\psi_t-\psi\psi_t^*\right)\mathrm{dt} \\ & +i\delta_4\int_{-\infty}^{\infty} t\,|\,\psi|^4\left(\psi^*\psi_t-\psi\psi_t^*\right)\mathrm{dt}+i\delta_6\int_{-\infty}^{\infty} t\,|\,\psi|^6\left(\psi^*\psi_t-\psi\psi_t^*\right)\mathrm{dt} \\ & +\frac{\beta_3}{2}\int_{-\infty}^{\infty}|\,\psi_t|^2\mathrm{dt}-\frac{3}{2}\gamma_s\int_{-\infty}^{\infty}|\,\psi|^4\mathrm{dt}-\frac{3}{2}\gamma_1\int_{-\infty}^{\infty}|\,\psi|^6\mathrm{dt} \\ & +2\delta_1\int_{-\infty}^{\infty} t\,|\,\psi|^4\mathrm{dt}-\frac{3}{2}\theta_1\int_{-\infty}^{\infty}|\,\psi|^8\mathrm{dt}. \end{aligned} \tag{11}$$

2.3. Width evolution

For the study of the evolution of the width, we refer to (5) and we have:

$$\frac{d\sigma}{\mathrm{dz}}=\frac{1}{2\sigma E}\int_{-\infty}^{\infty}(t-T)^2\left(\psi^*\psi_z+\psi\psi_z^*\right)\mathrm{dt}-\frac{\sigma}{2E}\frac{\mathrm{dE}}{\mathrm{dz}}-\frac{1}{\sigma E}\frac{\mathrm{dT}}{\mathrm{dz}}\int_{-\infty}^{\infty}(t-T)\,|\,\psi|^2\mathrm{dt}. \tag{12}$$

By inserting (1) in (12) we recover:

$$\frac{d\sigma}{\mathrm{dz}}=\frac{K}{2\sigma E}-\frac{\sigma}{2E}\frac{\mathrm{dE}}{\mathrm{dz}}-\frac{1}{\sigma E}\frac{\mathrm{dT}}{\mathrm{dz}}\int_{-\infty}^{\infty}(t-T)\,|\,\psi|^2\mathrm{dt} \tag{13}$$

where

$$\begin{aligned} K = \; & i\frac{\beta_2}{2}\int_{-\infty}^{\infty}(t-T)^2\left(\psi_{\mathrm{tt}}\psi^*-\psi_{\mathrm{tt}}^*\psi\right)\mathrm{dt}+i\frac{\alpha_3}{6}\int_{-\infty}^{\infty}(t-T)^2\left(\psi_t^*\psi_{\mathrm{tt}}-\psi_t\psi_{\mathrm{tt}}^*\right)\mathrm{dt} \\ & +\alpha_2\int_{-\infty}^{\infty}(t-T)^2\,|\,\psi_t|^2\mathrm{dt}+i\delta_2\int_{-\infty}^{\infty}(t-T)^2\,|\,\psi|^2\left(\psi^*\psi_t-\psi\psi_t^*\right)\mathrm{dt} \\ & +2\delta_3\int_{-\infty}^{\infty}(t-T)^2\,|\,\psi|^6\mathrm{dt}+i\delta_4\int_{-\infty}^{\infty}(t-T)^2\,|\,\psi|^4\left(\psi^*\psi_t-\psi\psi_t^*\right)\mathrm{dt} \\ & +2\delta_5\int_{-\infty}^{\infty}(t-T)^2\,|\,\psi|^8\mathrm{dt}+i\delta_6\int_{-\infty}^{\infty}(t-T)^2\,|\,\psi|^6\left(\psi^*\psi_t-\psi\psi_t^*\right)\mathrm{dt} \\ & +\frac{\beta_3}{6}\int_{-\infty}^{\infty}(t-T)^2\left(\psi_{\mathrm{ttt}}\psi^*+\psi_{\mathrm{ttt}}^*\psi\right)\mathrm{dt}+2\delta_1\int_{-\infty}^{\infty}(t-T)^2\,|\,\psi|^4\mathrm{dt} \\ & +\gamma_s\int_{-\infty}^{\infty}(t-T)^2\left[\left(|\psi|^2\psi\right)_t\psi^*+\left(|\psi|^2\psi^*\right)_t\psi\right]\mathrm{dt} \\ & +\gamma_1\int_{-\infty}^{\infty}(t-T)^2\left[\left(|\psi|^4\psi\right)_t\psi^*+\left(|\psi|^4\psi^*\right)_t\psi\right]\mathrm{dt} \\ & +\theta_1\int_{-\infty}^{\infty}(t-T)^2\left[\left(|\psi|^6\psi\right)_t\psi^*+\left(|\psi|^6\psi^*\right)_t\psi\right]\mathrm{dt}. \end{aligned} \tag{14}$$

2.4. Frequency variation

Using (4) we have:

$$\frac{d\Omega}{\mathrm{dz}}=i\frac{1}{2E}\int_{-\infty}^{\infty}\left[\left(\psi_z^*\psi_t-\psi_z\psi_t^*\right)+\left(\psi_{\mathrm{tz}}\psi^*-\psi\psi_{\mathrm{tz}}^*\right)\right]\mathrm{dt}-\frac{\Omega}{2E}\frac{\mathrm{dE}}{\mathrm{dz}}. \tag{15}$$

This expression may be transformed into the form:

$$\frac{d\Omega}{\mathrm{dz}}=i\frac{1}{2E}\int_{-\infty}^{\infty}(M+N)\mathrm{dt}-\frac{\Omega}{2E}\frac{\mathrm{dE}}{\mathrm{dz}} \tag{16}$$

where

$$
\begin{aligned}
N = \quad & -i\frac{\beta_2}{2}\left(\psi_{tt}\psi_t^* + \psi_{tt}^*\psi_t\right) - \frac{\alpha_2}{2}\left(\psi_{tt}\psi_t^* - \psi_{tt}^*\psi_t\right) - \frac{\beta_3}{6}\left(\psi_t\psi_{ttt}^* - \psi_t^*\psi_{ttt}\right) \\
& -i\frac{\alpha_3}{6}\left(\psi_t^*\psi_{ttt} + \psi_t\psi_{ttt}^*\right) + i\gamma_0 \mid \psi|^2\left(|\psi|^2\right)_t + i\gamma \mid \psi|^4\left(|\psi|^2\right)_t + i\theta \mid \psi|^2\left(|\psi|^2\right)_t \\
& +\delta_1 \mid \psi|^2\left(\psi^*\psi_t - \psi\psi_t^*\right) + \delta_3 \mid \psi|^4\left(\psi^*\psi_t - \psi\psi_t^*\right) + \delta_5 \mid \psi|^6\left(\psi^*\psi_t - \psi\psi_t^*\right) \\
& +\gamma_s\left[\left(|\psi|^2\psi^*\right)_t\psi_t - \left(|\psi|^2\psi\right)_t\psi_t^*\right] - i\delta_2\left[\left(|\psi|^2\psi^*\right)_t\psi_t + \left(|\psi|^2\psi\right)_t\psi_t^*\right] \\
& +\gamma_1\left[\left(|\psi|^4\psi^*\right)_t\psi_t - \left(|\psi|^4\psi\right)_t\psi_t^*\right] - i\delta_4\left[\left(|\psi|^4\psi^*\right)_t\psi_t + \left(|\psi|^4\psi\right)_t\psi_t^*\right] \\
& +\theta_1\left[\left(|\psi|^6\psi^*\right)_t\psi_t - \left(|\psi|^6\psi\right)_t\psi_t^*\right] - i\delta_6\left[\left(|\psi|^6\psi^*\right)_t\psi_t + \left(|\psi|^6\psi\right)_t\psi_t^*\right]
\end{aligned}
\tag{17}
$$

and

$$
\begin{aligned}
M = \quad & i\frac{\beta_2}{2}\left(\psi_{ttt}\psi^* + \psi_{ttt}^*\psi\right) - \frac{\alpha_2}{2}\left(\psi_{ttt}\psi^* - \psi_{ttt}^*\psi\right) - \frac{\beta_3}{6}\left(\psi_{tttt}\psi^* - \psi_{tttt}^*\psi\right) \\
& -i\frac{\alpha_3}{6}\left(\psi_{tttt}\psi^* + \psi_{tttt}^*\psi\right) - i\gamma_0\left[\left(|\psi|^2\psi\right)_t\psi^* + \left(|\psi|^2\psi^*\right)_t\psi\right] \\
& +\delta_1\left[\left(|\psi|^2\psi\right)_t\psi^* - \left(|\psi|^2\psi^*\right)_t\psi\right] + \gamma_s\left[\left(|\psi|^2\psi\right)_{tt}\psi^* - \left(|\psi|^2\psi^*\right)_{tt}\psi\right] \\
& +i\delta_2\left[\left(|\psi|^2\psi\right)_{tt}\psi^* + \left(|\psi|^2\psi^*\right)_{tt}\psi\right] - i\gamma\left[\left(|\psi|^4\psi\right)_t\psi^* + \left(|\psi|^4\psi^*\right)_t\psi\right] \\
& +\delta_3\left[\left(|\psi|^4\psi\right)_t\psi^* - \left(|\psi|^4\psi^*\right)_t\psi\right] + \gamma_1\left[\left(|\psi|^4\psi\right)_{tt}\psi^* - \left(|\psi|^4\psi^*\right)_{tt}\psi\right] \\
& +i\delta_4\left[\left(|\psi|^4\psi\right)_{tt}\psi^* + \left(|\psi|^4\psi^*\right)_{tt}\psi\right] - i\theta\left[\left(|\psi|^6\psi\right)_t\psi^* + \left(|\psi|^6\psi^*\right)_t\psi\right] \\
& +\delta_5\left[\left(|\psi|^6\psi\right)_t\psi^* - \left(|\psi|^6\psi^*\right)_t\psi\right] + \theta_1\left[\left(|\psi|^6\psi\right)_{tt}\psi^* - \left(|\psi|^6\psi^*\right)_{tt}\psi\right] \\
& +i\delta_6\left[\left(|\psi|^6\psi\right)_{tt}\psi^* + \left(|\psi|^6\psi^*\right)_{tt}\psi\right]
\end{aligned}
\tag{18}
$$

2.5. Chirp evolution

The variation of the chirp with respect to z is given by:

$$\frac{dC}{dz} = i\frac{1}{2E}\int_{-\infty}^{\infty}(t-T)\left[\left(\psi_z^*\psi_t - \psi_z\psi_t^*\right) + \left(\psi_{tz}\psi^* - \psi\psi_{tz}^*\right)\right]dt - \frac{C}{2E}\frac{dE}{dz} - \Omega\frac{dT}{dz}. \tag{19}$$

This can be put in the following form

$$\frac{dC}{dz} = i\frac{1}{2E}\int_{-\infty}^{\infty}(t-T)(M+N)dt - \frac{C}{2E}\frac{dE}{dz} - \Omega\frac{dT}{dz}. \tag{20}$$

3. Results and discussions

While using the moment method, the governing model is transformed to a dynamical system of parameters where the unknowns are T, σ, Ω, C and E. It is therefore necessary to model this electromagnetic pulse by a trial function characterized by frequency (Ω), chirp (C), shift (T) and width (σ). The trial function in today's work is the Gaussian function (Gaussons) given by:

$$\psi(z,t) = exp\left[-\left(\frac{t-T}{\sigma}\right)^2 + i\left(C(t-T)^2 + \Omega(t-T)\right)\right] \tag{21}$$

The variational equations are:

(22) Energy variational equation

Table 1
NLSE parameters values.

Parameters	Values	Parameters	Values
β_2	$1.7674 * 10^{-23} s^2/m$	γ	$-1.6600 * 10^{-30} m^3 V^{-4}$
α_2	$1.7674 * 10^{-23} s^2/m$	δ_3	$6.6278 * 10^{-31} m^3 V^{-4}$
β_3	$-3.488 * 10^{-38} s^3/m$	γ_1	$-1.0920 * 10^{-45} m^3 V^{-4} s^{-1}$
α_3	$-3.4707 * 10^{-38} s^3/m$	δ_4	$4.36 * 10^{-46} m^3 V^{-4} s^{-1}$
γ_0	$-4.7655 * 10^{-12} m V^{-2}$	θ	$-1.8457 * 10^{-49} m^5 V^{-6}$
δ_1	$2.3301 * 10^{-12} m V^{-2}$	δ_5	$6.3431 * 10^{-50} m^5 V^{-6}$
γ_s	$-3.1349 * 10^{-27} m V^{-2} s^{-1}$	θ_1	$-1.2142 * 10^{-64} m^5 V^{-6} s^{-1}$
δ_2	$1.5328 * 10^{-27} m V^{-2} s^{-1}$	δ_6	$4.1727 * 10^{-65} m^5 V^{-6} s^{-1}$

$$\begin{aligned}\frac{\mathrm{dE}}{\mathrm{dz}} = & \frac{\alpha_2}{2\sigma}\left(\sigma^2\Omega^2 + \sigma^4 C^2 + 1\right)\sqrt{2\pi} - \frac{\alpha_3\Omega}{6\sigma}\left(\sigma^2\Omega^2 + 3\sigma^4 C^2 + 3\right)\sqrt{2\pi} \\ & + \left(\frac{\delta_1\sqrt{2}}{2} + \frac{\delta_3\sqrt{3}}{3} + \frac{\delta_5}{2}\right)\sigma\sqrt{2\pi} + \left(\delta_2 + \frac{\delta_4}{\sqrt{2}} + \frac{\delta_6\sqrt{3}}{3}\right)\sigma\Omega\sqrt{2\pi}\end{aligned} \tag{22}$$

(23) Shift variational equation

$$\begin{aligned}\frac{\mathrm{dT}}{\mathrm{dz}} = & -\beta_2\Omega + \alpha_2 C\Omega\sigma^2 - \frac{\alpha_3}{2}\left(\sigma^4 C^3 + \sigma^2 C\Omega^2 + 3\Omega\right) + \frac{\beta_3}{2}\left(\Omega^2 + \sigma^2 C^2 + \frac{1}{\sigma^2}\right) \\ & - \frac{3\sqrt{2}}{4}\gamma_s - \frac{\sqrt{3}}{2}\gamma_1 - \frac{3}{4}\theta_1 - \delta_2\left[\frac{\sqrt{2}}{4}\sigma^2 C - \left(2 - \sqrt{2}\right)T\Omega\right] \\ & - \delta_4\left[\frac{\sqrt{3}}{9}\sigma^2 C - \left(\sqrt{2} - \frac{2\sqrt{3}}{3}\right)T\Omega\right] - \delta_6\left[\frac{1}{8}\sigma^2 C - \left(\frac{2\sqrt{3}}{3} - 1\right)T\Omega\right]\end{aligned} \tag{23}$$

(24) Width variational equation

$$\begin{aligned}\frac{d\sigma}{\mathrm{dz}} = & \frac{\beta_2}{2}C\Omega + \alpha_2\left(\frac{7}{8}C^2\sigma^3 - \frac{3}{8}\Omega^2\sigma - \frac{1}{8\sigma}\right) + \alpha_3\left(\frac{3}{24}C^2\sigma^3\Omega + \frac{3}{24}\Omega^3\sigma - \frac{7\Omega}{24\sigma}\right) \\ & - \left[\frac{(\sqrt{2} - 16)}{16}\delta_2 + \frac{(\sqrt{6} - 36)}{36\sqrt{2}}\delta_4 + \frac{(3 - 32\sqrt{3})}{96}\delta_6\right]\sigma\Omega \\ & - \left(\frac{7\delta_1\sqrt{2}}{16} + \frac{11\delta_3\sqrt{3}}{36} + \frac{15\delta_5}{32}\right)\sigma\end{aligned} \tag{24}$$

(25) Frequency variational equation

$$\begin{aligned}\frac{d\Omega}{\mathrm{dz}} = & \frac{\alpha_2\Omega}{2\sigma^2}\left(7C^2\sigma^4 + 3\sigma^2\Omega^2 + 7\right) - \left(\delta_2 + \frac{\delta_4}{\sqrt{2}} + \frac{\delta_6\sqrt{3}}{3}\right)\Omega^2 \\ & + \frac{\alpha_3}{6\sigma^4}\left(\sigma^4\Omega^4 + 12\Omega^2\sigma^2 + 6C^4\sigma^8 + 8C^2\Omega^2\sigma^6 + 18C^2\sigma^4 + 12\right) \\ & + \left(\sqrt{2}\gamma_s + \frac{8\sqrt{3}}{9}\gamma_1 + \frac{3}{2}\theta_1\right)C - 3\left(\frac{\delta_1\sqrt{2}}{2} + \frac{\delta_3\sqrt{3}}{3} + \frac{\delta_5}{2}\right)\Omega\end{aligned} \tag{25}$$

(26) Chirp variational equation

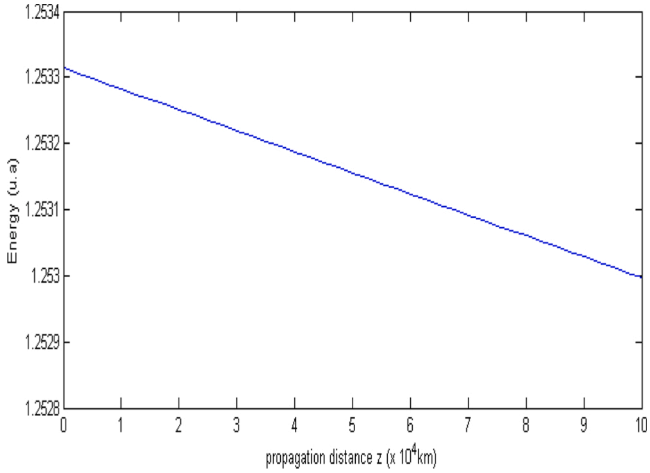

Fig. 1. The evolution of the energy with respect to the propagation distance z.

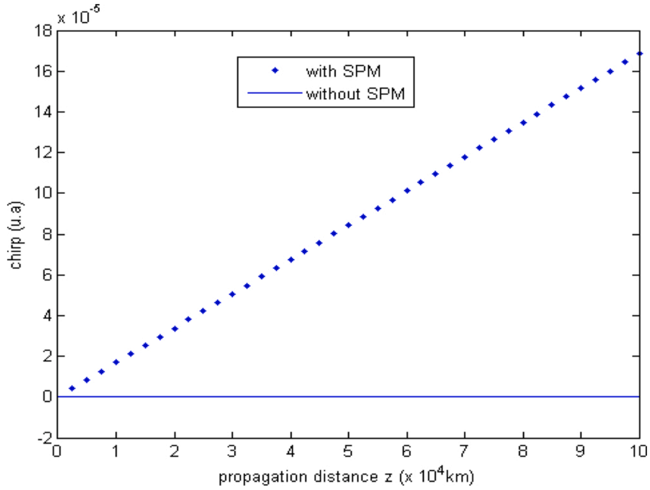

Fig. 2. The evolution of the chirp with respect to the propagation distance z.

$$
\begin{aligned}
\frac{\mathrm{dC}}{\mathrm{dz}} = \; & -\frac{\beta_2}{\sigma^2}\left(1+C^2\sigma^4\right)-\frac{\alpha_2 C}{2\sigma^2}\left(C^2\sigma^6+2\sigma^4C^2+10\Omega^2\sigma^4+2\Omega^2\sigma^2+3\sigma^2+2\right) \\
& +\frac{\beta_3\Omega}{\sigma^2}\left(1+\sigma^4C^2\right)+\alpha_3\left(C^3\sigma^2\Omega+\frac{2}{3}C\Omega^3+\frac{3}{2}\Omega^2-2C\Omega-\frac{1}{6}\Omega^3C\sigma^2-\frac{3}{2}C^3\Omega\sigma^4\right) \\
& -\frac{\delta_1 C\sqrt{2}}{4}\left(4+\sigma^2\right)-\frac{\delta_3 C\sqrt{3}}{9}\left(6+\sqrt{2}\,\sigma^2\right)-\frac{\delta_5 C}{8}\left(8+\sigma^2\right) \\
& +\delta_2\left[\left(\frac{3\sqrt{2}}{4}\sigma^2-2\right)C-\left(2-\sqrt{2}\right)T\Omega\right]\Omega+\left(\frac{7\sqrt{2}}{4}\gamma_s+\frac{23\sqrt{3}}{18}\gamma_1+2\theta_1\right)\Omega \\
& +\delta_4\left[\left(\frac{3\sqrt{3}}{9}\sigma^2-\sqrt{2}\right)C-\left(\sqrt{2}-\frac{2\sqrt{3}}{3}\right)T\Omega\right]\Omega-\left(\frac{20\sqrt{3}-9}{72}\right)\gamma-\left(\frac{21-2\sqrt{6}}{48}\right)\theta \\
& -\frac{\sqrt{2}}{4}\gamma_0+\delta_6\left[\left(\frac{3}{8}\sigma^2-\frac{2\sqrt{3}}{3}\right)C-\left(\frac{2\sqrt{3}}{3}-1\right)T\Omega\right]\Omega.
\end{aligned}
\tag{26}
$$

Using the value of the NLSE parameters (see Table 1) and Runge–Kutta algorithm (RK4) in matlab, we obtain these figures:

From Eq. (22), we can say that the physical system studied is not conservative. Thus the parameters of gain and absorption of NLSE which are linked to the imaginary parts of the refractive index in influence the conservation of the energy of the pulse. Then, the

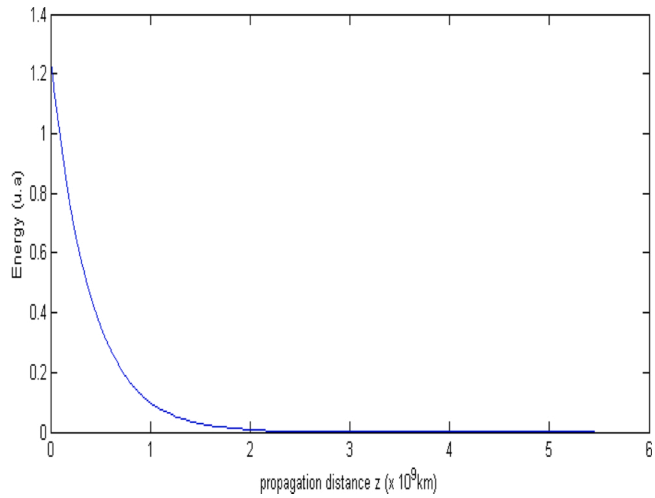

Fig. 3. The evolution of the energy with respect to the propagation distance z.

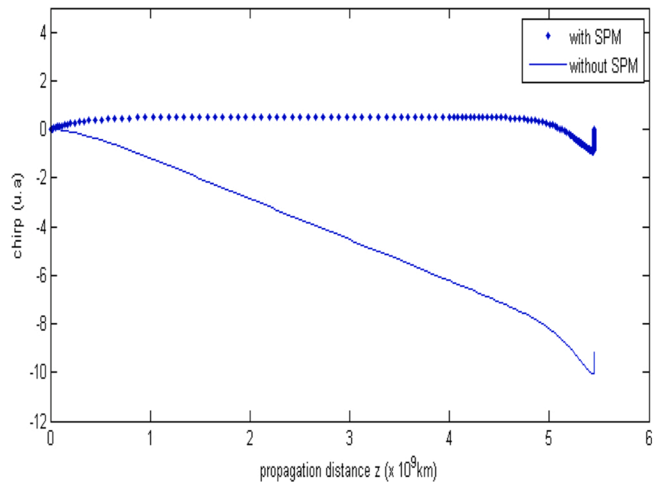

Fig. 4. The evolution of the chirp with respect to the propagation distance z.

waveguide based on ENZ-materials which will minimize the energy losses will be designed by not only reducing the values of the permittivity of the basic ENZ-material but also by removing the imaginary part of this refractive index. When we refer to Figs. 1 and 3, we note that the energy decreases during the propagation of the pulse and the energy curves are not linear. This decrease is very small (in this case the loss energy is 0.025%). This is explained by the low value of the permittivity of the ENZ–materials. As regards the phenomenon of self-phase modulation, it only affects the phase of the pulse, precisely its chirp. It manifests itself here by an increase in the chirp during the propagation of the pulse (Figs. 2 and 4). Using Figs. 3 and 4, we remark that the pulse studied is not defined beyond 5.4×10^8 km. Finally, Fig. 4 shows edge effects of the chirp. The appearance of these effects is explained by the absence of periodic boundary conditions.

4. Conclusions

In this work, we obtained from the moment method, the variational dynamics of the characteristic parameters of Gausson pulses in a waveguide based on an ENZ-material. The model showed that the system studied is dissipative. But the energy loss is negligible due to a low value of the permittivity of ENZ-materials. The effect of self–phase modulation results in the increase of the chirp during the propagation of the Gausson pulse. In future, one of the several avenues is to use anisotropic property and birefringent characteristics of certain ENZ-materials to study logarithmic nonlinearity in waveguides based on ENZ-materials. The results will be disseminated, with time.

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The authors also declare that there is no conflict of interest.

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