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RESEARCH ARTICLE

A numerical method for the Hirota equation in a dispersive optical media

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Abstract In this study, the propagation of ultrashort optical pulses in the context of long-distance optical fiber communications is numerically investigated. The method used is the finite difference scheme in the third order time domain and periodic boundary conditions. As a result, the obtained discrete system of ordinary differential equations is solved numerically by the fourth-order Runge–Kutta algorithm. The proposed algorithm was tested on various input pulses. Precise results of temporal mappings are presented.

Keywords Ultrashort pulse · Optical fiber propagation · Finite difference time-domain method · Periodic boundary conditions

Introduction

Over the past decade, the theory of pulse propagation in optical fibers and several other materials has been widely studied [1–8]. It appears that the nonlinear Schrödinger equation is essential to describe optical propagation in various fields such as the field of nonlinear optics, optical fiber communications, etc [9–18]. In the recent era of computer networks and telecommunications, the field of research on soliton theory and their applications in optical fiber is becoming more and more crucial. Since then, different models of the Schrödinger equation have succeeded in solving the challenges of the dynamics of the propagation of solitons pulses in fibers for high-speed transmission needs, thus leading to considerable technological improvement [19–25]. In the literature, several theoretical and experimental works have been the subject of intensive research [26–33]. Since then, significant progress has been made in the development of various schemes for processing the nonlinear Schrödinger equation. For approximate schemes, we cite the Adomian decomposition [34–38], the complete discriminant approach [39, 40], the Csch technique and the sine-cosine strategy [21], the Lie analysis method [41]. Although constructing an exact analytical solution is more important as it can provide the best understanding of the nature of the model to be addressed effectively, researchers have developed various powerful tools to analyze the Schrödinger equation. These techniques include the Kudryashov approach [42–45], the method of mapping generalized Riccati equations [46–49], Ansatz soliton method [47], the method of Jacobi elliptic functions [50], the tanh-coth method [51].

Due to the nonlinearity of the Schrödinger equation, a numerical approach is often necessary to understand the dynamics of pulse propagation in optical fibers. In the literature, many numerical methods are used and the

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most popular ones are: fractional step method, spectral methods, finite difference method in the time domain, etc. One of the most common challenges in developing these numerical schemes is that they satisfy conservation laws [52]. Based on different discretization techniques, several differential numerical methods in the time domain have effectively simulated nonlinear dispersive media. These approaches include linear and nonlinear properties and take into account the physical complexity of media such as Kerr effects, Raman scattering and/or photon absorpt. In this paper, we have solved the third-order nonlinear Schrödinger equation (TONSE) used to describe the propagation of ultrashort pulses in optical fibers using a third-order finite-difference time-domain scheme with conditions at periodic limits. The importance of using periodic boundary conditions is to avoid discontinuity problems that could arise during numerical processing [3]. The fourth order Runge–Kutta algorithm is applied to the obtained system of ordinary differential equations. This present work is organized as follows: the governance model is first described using Third-Order Nonlinear Schrödinger Equation (TONSE). Then, the finite difference method is presented to solve this equation. The last section is devoted to numerical simulations to present the mapping of the temporal evolution of the pulses during their propagation along the fiber.

Governing model

We consider a third-order nonlinear Schrödinger equation (TONSE) used to describe the propagation of an ultrashort optical pulse in the context of long-distance communications over optical fiber transmission lines with the following form [12]:

$$i\,U_z + \alpha_2(U_{tt} + 2U|U|^2) + i\alpha_3(U_{ttt} + 6\,U_t|U|^2) = 0 \tag{1}$$

where $U = U(t,z)$ is the slowly varying complex pulse envelope. This Eq. (1) called Hirota equation can be used to describe the propagation of subpicosecond or femtosecond optical pulse in fibers thanks to it taking into account higher-order dispersion and time-delay corrections to the cubic nonlinearity. In this equation,

- t is the retarded time variable with $t \in]-T, T[$ and $T > 0$,
- z is the propagation variable in a moving frame with $z \in \mathbf{R}_+^*$,
- U_{ttt} is the third-order dispersion term,
- $U_t|U|^2$ is the time-delay correction to the cubic term.

When $\alpha_2 = 1$ and $\alpha_3 = 0$, the Eq. (1) reduces to the NLS equation. While $\alpha_2 = 0$ and $\alpha_3 = 1$, the Hirota equation reduces to modified Korteweg-de Vries equation.

U is a function given in Hilbert space and the periodic boundary conditions used are:

$$\begin{cases} U(-T,z) = U(T,z) & z > 0 \\ \dfrac{\partial}{\partial z}U(-T,z) = \dfrac{\partial}{\partial z}U(T,z) & z > 0 \\ \dfrac{\partial^2}{\partial z^2}U(-T,z) = \dfrac{\partial^2}{\partial z^2}U(T,z) & z > 0 \end{cases} \tag{2}$$

Finite-difference time-domain scheme: method details

The formalism principle used is presented as follows: let's consider a uniform grid $t_{j_{1\le j\le N}}$ step h in t. Let $U_j(z)$ an approximation of $U(t_j\,z)$ for all $z \in]0, L[$ with L is the fiber length and $j = 1, 2, ..., J$. Let's use approximation with eight points $\left(t_k, z\right)_{k=j-4}^{j+3}$ to express the partial derivatives $\frac{\partial U}{\partial t}\left(t_j, z\right)$,$\frac{\partial^2 U}{\partial t^2}\left(t_j, z\right)$, $\frac{\partial^3 U}{\partial t^3}\left(t_j, z\right)$.

Let's expand the partial derivatives in the following form:

$$\begin{aligned} R_{j,n}Uz = {} & aU_{j-4}(z) + bU_{j-3}(z) + cU_{j-2}(z) \\ & + dU_{j-1}(z) + eU_j(z) + fU_{j+1}(z) \\ & + gU_{j+2}(z) + kU_{j+3}(z) \end{aligned} \tag{3}$$

such that for fixed $r \in 1,2,3$ we have:

$$R_{j,n}Uz = \frac{\partial^n U}{\partial t^n}\left(t_j, z\right) + O(h^{8-n}) \tag{4}$$

This accuracy is desired heuristically that the leading term of the truncation error be $\frac{\partial^n U}{\partial t^n}\left(t_j, z\right)$ and does not bring a parasite dispersion. Assuming that U is regular and by the Taylor expansion up to $8th$ order of $R_{j,n}Uz$ at the $\left(t_j, z\right)$, we have:

$$\begin{aligned}R_{j,n}Uz &= (a+b+c+d+e+f+g+k)U\left(t_j,z\right)\\ &+h(-4a-3b-2c-d+f+2g+3k)\frac{\partial U}{\partial t}\left(t_j,z\right)\\ &+\frac{h^2}{2!}(16a+9b+4c+d+f+4g+9k)\frac{\partial^2 U}{\partial t^2}\left(t_j,z\right)\\ &+\frac{h^3}{3!}(-64a-27b-8c-d+f+8g+27k)\frac{\partial^3 U}{\partial t^3}\left(t_j,z\right)\\ &+\frac{h^4}{4!}(256a+81b+16c+d+f+16g+81k)\frac{\partial^4 U}{\partial t^4}\left(t_j,z\right)\\ &+\frac{h^5}{5!}(-1024a-243b-32c-d+f+32g+243k)\frac{\partial^5 U}{\partial t^5}\left(t_j,z\right)\\ &+\frac{h^6}{6!}(4096a+729b+64c+d+f+64g+729k)\frac{\partial^6 U}{\partial t^6}\left(t_j,z\right)\\ &+\frac{h^7}{7!}(-16384a-2187b-128c-d+f+128g+2187k)\frac{\partial^7 U}{\partial t^7}\left(t_j,z\right)\\ &+\frac{h^8}{8!}(65536a+6561b+256c+d+f+256g+6561k)\frac{\partial^8 U}{\partial t^8}\left(t_j,z\right)\end{aligned} \tag{5}$$

Eq. (4) becomes:

$$\begin{cases} a+b+c+d+e+f+g+k &= (0,0,0)\\ -4a-3b-2c-d+f+2g+3k &= \left(\frac{1}{h},0,0\right)\\ 16a+9b+4c+d+f+4g+9k &= \left(0,\frac{2}{h^2},0\right)\\ -64a-27b-8c-d+f+8g+27k &= \left(0,0,\frac{6}{h^3}\right)\\ 256a+81b+16c+d+f+16g+81k &= (0,0,0)\\ -1024a-243b-32c-d+f+32g+243k &= (0,0,0)\\ 4096a+729b+64c+d+f+64g+729k &= (0,0,0)\\ -16384a-2187b-128c-d+f+128g+2187k &= (0,0,0)\end{cases} \tag{6}$$

Solving this system of equations we obtain:

$$\begin{cases} a &= \left(\frac{1}{140h},0,-\frac{7}{120h^3}\right)\\ b &= \left(-\frac{1}{15h},\frac{1}{90h^2},\frac{8}{15h^3}\right)\\ c &= \left(\frac{3}{10h},-\frac{3}{20h^2},-\frac{89}{40h^3}\right)\\ d &= \left(-\frac{1}{h},\frac{3}{2h^2},\frac{11}{3h^3}\right)\\ e &= \left(\frac{1}{4h},-\frac{49}{18h^2},-\frac{49}{24h^3}\right)\\ f &= \left(\frac{3}{5h},\frac{3}{2h^2},-\frac{2}{5h^3}\right)\\ g &= \left(-\frac{1}{10h},-\frac{3}{20h^2},\frac{71}{120h^3}\right)\\ k &= \left(\frac{1}{105h},\frac{1}{90h^2},-\frac{1}{15h^3}\right)\end{cases} \tag{7}$$

One can deduce that

$$65536a+6561b+256c+d+f+256g+6561k=\left(\frac{144}{h},\frac{72}{h^2},-\frac{1176}{h^6}\right) \tag{8}$$

then the leading term of the truncation error is:

$$E_P = \left(\frac{h^7}{280}, \frac{h^6}{560}, -\frac{7h^5}{240}, \frac{\partial^8 U}{\partial t^8}\left(t_j, z\right) \right) \tag{9}$$

We have:

$$\left\{ \begin{aligned} \frac{\partial U}{\partial t}\left(t_j, z\right) &= \frac{1}{420h}\Big(3U_{j-4}(z) - 28U_{j-3}(z) + 126U_{j-2}(z) - 420\, U_{j-1}(z) \\ &\quad + 105U_j(z) + 252U_{j+1}(z) - 42U_{j+2}(z) + 4U_{j+3}(z)\Big) - \frac{h^7}{280}\frac{\partial^8 U}{\partial t^8}\left(t_j, z\right) + \ldots \\ \frac{\partial^2 U}{\partial t^2}\left(t_j, z\right) &= \frac{1}{180h^2}\Big(2U_{j-3}(z) - 27U_{j-2}(z) + 270\, U_{j-1}(z) \\ &\quad - 490U_j(z) + 270U_{j+1}(z) - 27U_{j+2}(z) + 2U_{j+3}(z)\Big) - \frac{h^6}{560}\frac{\partial^8 U}{\partial t^8}\left(t_j, z\right) + \ldots \\ \frac{\partial^3 U}{\partial t^3}\left(t_j, z\right) &= \frac{1}{120h^3}\Big(-7U_{j-4}(z) + 64U_{j-3}(z) - 267U_{j-2}(z) + 440\, U_{j-1}(z) \\ &\quad - 245U_j(z) - 48U_{j+1}(z) + 71U_{j+2}(z) - 8U_{j+3}(z)\Big) + \frac{7h^5}{240}\frac{\partial^8 U}{\partial t^8}\left(t_j, z\right) + \ldots \end{aligned} \right. \tag{10}$$

In condensed form the previous system is equivalent to the following system:

$$\left\{ \begin{aligned} \frac{\partial U}{\partial t}\left(t_j, z\right) &= R_{j,1}Uz - \frac{h^7}{280}\frac{\partial^8 U}{\partial t^8}\left(t_j, z\right) + \ldots \\ \frac{\partial^2 U}{\partial t^2}\left(t_j, z\right) &= R_{j,2}Uz - \frac{h^6}{560}\frac{\partial^8 U}{\partial t^8}\left(t_j, z\right) + \ldots \\ \frac{\partial^3 U}{\partial t^3}\left(t_j, z\right) &= R_{j,3}Uz + \frac{7h^5}{240}\frac{\partial^8 U}{\partial t^8}\left(t_j, z\right) + \ldots \end{aligned} \right. \tag{11}$$

It follows that the system of ordinary differential equations:

$$\frac{dU_j}{dz} = i\alpha_2 R_{j,2}Uz + 2i\alpha_2 U_j\left|U_j\right|^2 - \alpha_3 R_{j,3}Uz - 6\alpha_3 R_{j,1}Uz\left|U_j\right|^2 \tag{12}$$

This equation must be cast into the form:

$$\begin{aligned} \frac{dU_j}{dz} &= \frac{i\,\alpha_2}{180h^2}\Big[2U_{j-3}(z) - 27U_{j-2}(z) + 270\, U_{j-1}(z) \\ &\quad - 490U_j(z) + 270U_{j+1}(z) - 27U_{j+2}(z) + 2U_{j+3}(z)\Big] \\ &\quad - \frac{\alpha_3}{120h^3}\Big[-7U_{j-4}(z) + 64U_{j-3}(z) - 267U_{j-2}(z) + 440\, U_{j-1}(z) \\ &\quad - 245U_j(z) - 48U_{j+1}(z) + 71U_{j+2}(z) - 8U_{j+3}(z)\Big] \\ &\quad - \frac{6\,\alpha_3}{420h}\Big[3U_{j-4}(z) - 28U_{j-3}(z) + 126U_{j-2}(z) - 420\, U_{j-1}(z) \\ &\quad + 105U_j(z) + 252U_{j+1}(z) - 42U_{j+2}(z) + 4U_{j+3}(z)\Big]\left|U_j(z)\right|^2 \\ &\quad + 2i\,\alpha_2\Big[U_j(z)\left|U_j(z)\right|^2\Big] \end{aligned} \tag{13}$$

$\forall\, z \in\,]0, L[$ and $j = 5, \ldots, J-3$

Though periodic boundary conditions, the system completed by three following Eqs. (14),(15),(16) and (17).

$\forall\, z \in\,]0, L[$,

$$\begin{aligned} \frac{dU_1}{dz} &= \frac{i\,\alpha_2}{180h^2}\Big[2U_{J-3}(z) - 27U_{J-2}(z) + 270\, U_{J-1}(z) \\ &\quad - 490U_1(z) + 270U_2(z) - 27U_3(z) + 2U_4(z)\Big] \\ &\quad - \frac{\alpha_3}{120h^3}\Big[-7U_{J-4}(z) + 64U_{J-3}(z) - 267U_{J-2}(z) + 440\, U_{J-1}(z) \\ &\quad - 245U_1(z) - 48U_2(z) + 71U_3(z) - 8U_4(z)\Big] \\ &\quad - \frac{6\,\alpha_3}{420h}\Big[3U_{J-4}(z) - 28U_{J-3}(z) + 126U_{J-2}(z) - 420\, U_{J-1}(z) \\ &\quad + 105U_1(z) + 252U_2(z) - 42U_3(z) + 4U_4(z)\Big]\left|U_1(z)\right|^2 \\ &\quad + 2i\,\alpha_2\Big[U_1(z)\left|U_1(z)\right|^2\Big] \end{aligned} \tag{14}$$

$$\begin{aligned} \frac{dU_2}{dz} &= \frac{i\,\alpha_2}{180h^2}\Big[2U_{J-2}(z) - 27U_{J-1}(z) + 270\, U_1(z) \\ &\quad - 490U_2(z) + 270U_3(z) - 27U_4(z) + 2U_5(z)\Big] \\ &\quad - \frac{\alpha_3}{120h^3}\Big[-7U_{J-3}(z) + 64U_{J-2}(z) - 267U_{J-1}(z) + 440\, U_1(z) \\ &\quad - 245U_2(z) - 48U_3(z) + 71U_4(z) - 8U_5(z)\Big] \\ &\quad - \frac{6\,\alpha_3}{420h}\Big[3U_{J-3}(z) - 28U_{J-2}(z) + 126U_{J-1}(z) - 420\, U_1(z) \\ &\quad + 105U_2(z) + 252U_3(z) - 42U_4(z) + 4U_5(z)\Big]\left|U_2(z)\right|^2 \\ &\quad + 2i\,\alpha_2\Big[U_2(z)\left|U_2(z)\right|^2\Big] \end{aligned} \tag{15}$$

$$\frac{dU_3}{dz} = \frac{i\,\alpha_2}{180h^2}\Big[2U_{J-1}(z) - 27U_1(z) + 270\,U_2(z) - 490U_3(z) + 270U_4(z) - 27U_5(z) + 2U_6(z)\Big] - \frac{\alpha_3}{120h^3}\Big[-7U_{J-2}(z) + 64U_{J-1}(z) - 267U_1(z) + 440\,U_2(z) - 245U_3(z) - 48U_4(z) + 71U_5(z) - 8U_6(z)\Big] - \frac{6\,\alpha_3}{420h}\Big[3U_{J-2}(z) - 28U_{J-1}(z) + 126U_1(z) - 420\,U_2(z) + 105U_3(z) + 252U_4(z) - 42U_5(z) + 4U_6(z)\Big]|U_3(z)|^2 + 2i\,\alpha_2\Big[U_3(z)\big|U_3(z)\big|^2\Big] \tag{16}$$

$$\frac{dU_{J-1}}{dz} = \frac{i\,\alpha_2}{180h^2}\Big[2U_{J-4}(z) - 27U_{J-3}(z) + 270\,U_{J-2}(z) - 490U_{J-1}(z) + 270U_1(z) - 27U_2(z) + 2U_3(z)\Big] - \frac{\alpha_3}{120h^3}\Big[-7U_{J-5}(z) + 64U_{J-4}(z) - 267U_{J-3}(z) + 440\,U_{J-2}(z) - 245U_{J-1}(z) - 48U_1(z) + 71U_2(z) - 8U_3(z)\Big] - \frac{6\,\alpha_3}{420h}\Big[3U_{J-5}(z) - 28U_{J-4}(z) + 126U_{J-3}(z) - 420\,U_{J-2}(z) + 105U_{J-1}(z) + 252U_1(z) - 42U_2(z) + 4U_3(z)\Big]\big|U_{J-1}(z)\big|^2 + 2i\,\alpha_2\Big[U_{J-1}(z)\big|U_{J-1}(z)\big|^2\Big] \tag{17}$$

Fourth order Runge–Kutta method for spatial integration

This integration will be done by fourth order of classical Runge–Kutta method that will give anyway a precision in $O(\triangle t^4)$.

The system of ordinary differential equations is:

For $j = 1, ..., J$

$$\frac{dU_j}{dz} = i\alpha_2 R_{j,2}Uz + 2i\alpha_2 U_j\big|U_j\big|^2 - \alpha_3 R_{j,3}Uz - 6\alpha_3 R_{j,1}Uz\big|U_j\big|^2 \tag{18}$$

Let's denote

$$U(z) = U_j(z)_{j=1}^{J} \tag{19}$$

and

$$\check{}(z) = L_j(U(z))_{j=1}^{J} \tag{20}$$

where

$$\frac{dU_j}{dz} = i\alpha_2 R_{j,2}Uz + 2i\alpha_2 U_j\big|U_j\big|^2 - \alpha_3 R_{j,3}Uz - 6\alpha_3 R_{j,1}Uz\big|U_j\big|^2 \tag{21}$$

We'll deal with Cauchy problem:

$$\begin{cases} \dfrac{dU}{dz} = \check{}(U) \\ U(0) = f(t_j)_{j=1}^{J} \end{cases} \tag{22}$$

The classic fourth order Runge–Kutta method applied to the obtained system of ordinary equations can be put in the form:

$$U(z + \Delta z) = U(z) + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) \tag{23}$$

with

$$\begin{cases} K_1 = \Delta z\check{}(U(z)) \\ K_2 = \Delta z\check{}\Big(U(z) + \dfrac{1}{2}K_1\Big) \\ K_3 = \Delta z\check{}\Big(U(z) + \dfrac{1}{2}K_2\Big) \\ K_4 = \Delta z\check{}\big(U(z) + K_3\big) \end{cases} \tag{24}$$

Results and discussion

We present the results of the two different cases to illustrate the accuracy and stability of the proposed scheme.

Scenario 1 $\alpha_2 = 1$ and $\alpha_3 = 0$, the Eq. (1) reduces to the NLS equation:

$$\begin{cases} i\,U_z + U_{tt} + 2U|U|^2 = 0 \\ U(t, z = 0) = U_0\exp(-t^2) \end{cases} \tag{25}$$

In order to validate the finite difference scheme developed, a first experiment consisted of presenting the evolution of a Gaussian pulse along an optical fiber governed by the NLS of order 2. Figure 1 presents on the one hand the the temporal evolution at distinct propagation distances z and on the other hand the temporal mapping of the evolution along the fiber. The results obtained are the typical results of propagation of a Gaussian signal along a fiber governed by the Eq. (1). This evolution has been widely studied in the literature [19].

In order to evaluate the impact the third order dispersion (TOD) and higher order nonlinear effects on ultrashort pulses, we studied the two scenarios below:

Scenario 2 $\alpha_2 = 0.6$ and $\alpha_3 = 0.25$, the Eq. (1) becomes:

$$\begin{cases} i\,U_z + \alpha_2(U_{tt} + 2U|U|^2) + i\alpha_3(U_{ttt} + 6\,U_t|U|^2) = 0 \\ U(t, z = 0) = U_0\cosh(t/5)\exp(-t^2/4). \end{cases} \tag{26}$$

Scenario 3 $\alpha_2 = 0$ and $\alpha_3 = 1$, the Eq. (1) becomes:

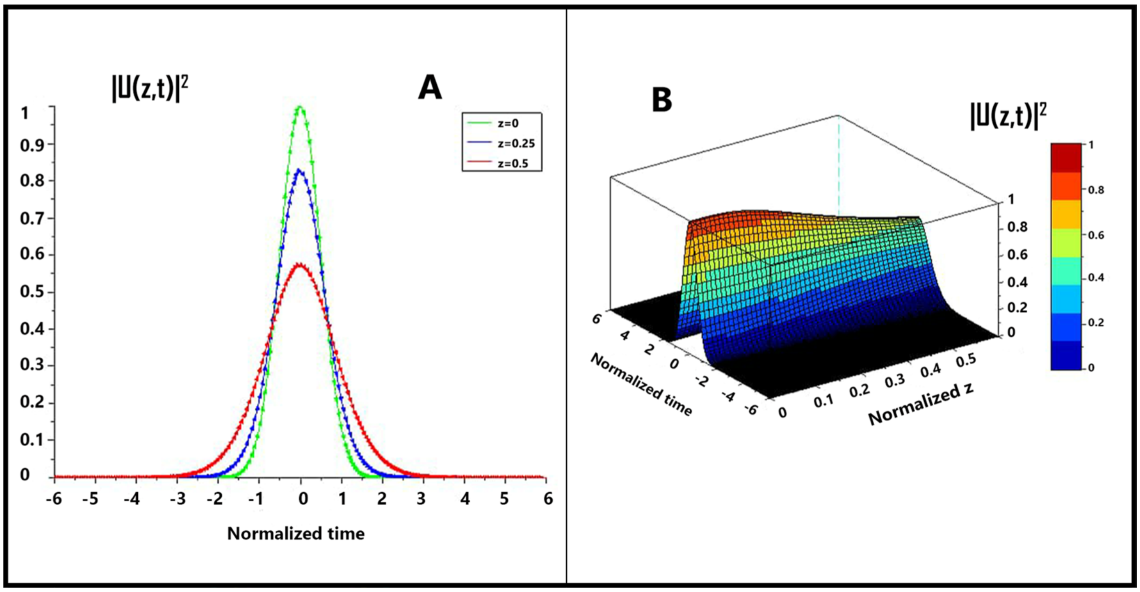

Fig. 1 The temporal evolution at distinct distances and along the fiber

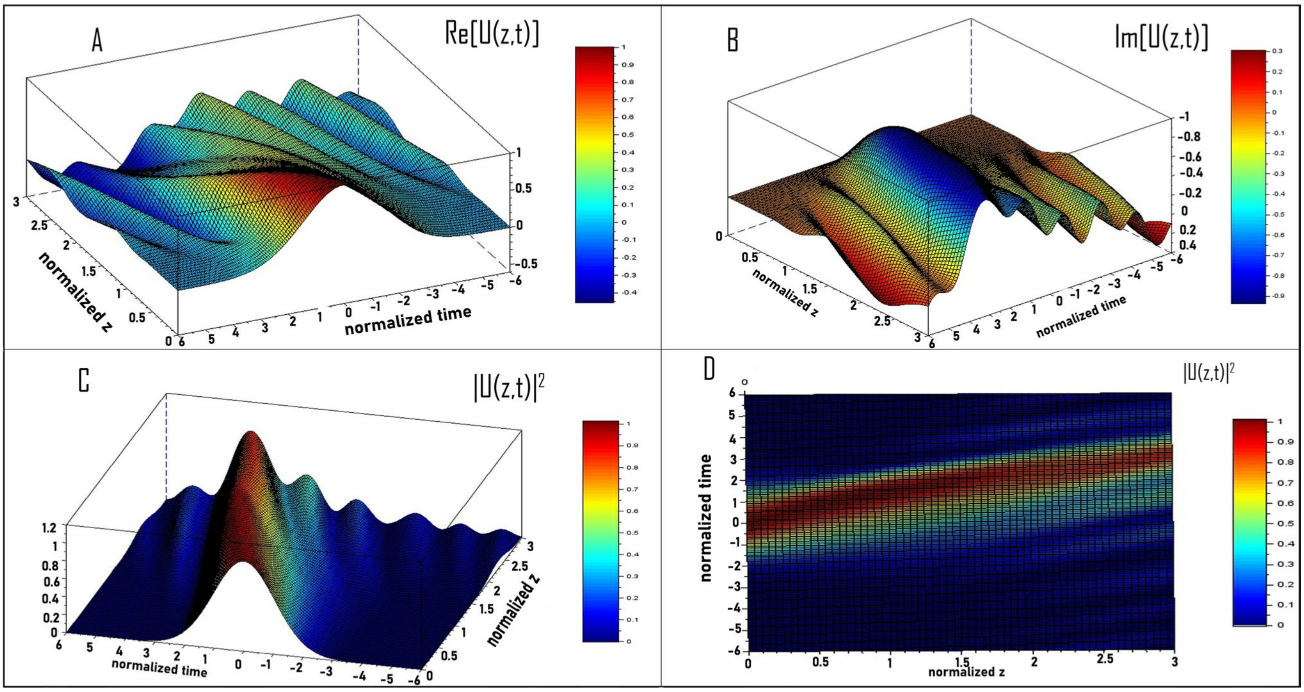

Fig. 2 **a** Temporal evolution of the real part of the amplitude, **b** temporal evolution of the imaginary part of the amplitude, **c** temporal evolution of $|U|^2$, **d** contour plot of $|U|^2$ as a function of the propagation distance along the fiber in the scenario 2 ($\alpha_2 = 0.6$ and $\alpha_3 = 0.25$)

$$\begin{cases} U_z + U_{ttt} + 6\,U_t|U|^2 = 0 \\ U(t,z=0) = U_0\cosh(t/5)\exp(-t^2/4). \end{cases} \tag{27}$$

The cartography of the temporal evolution of the real and imaginary part as well as $|U|^2$ are shown in Figs. 2 and 3 regarding propagation distance along the optical fiber.

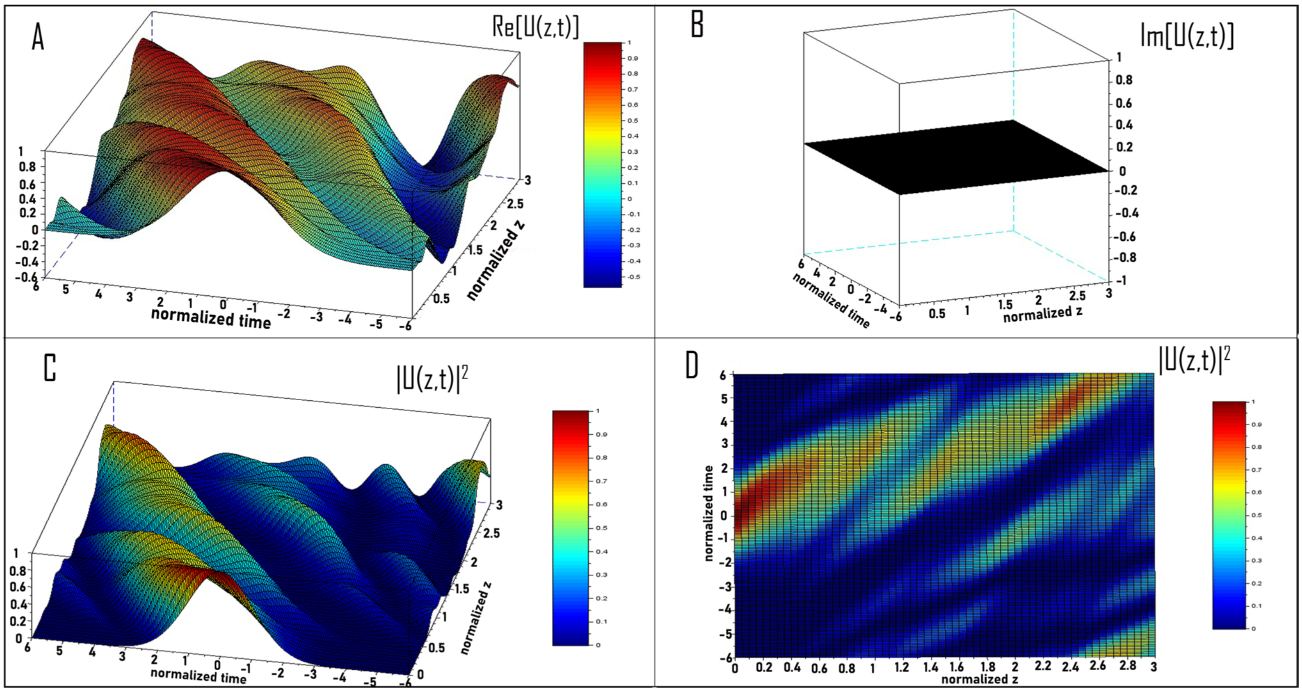

Fig. 3 **a** Temporal evolution of the real part of the amplitude, **b** temporal evolution of the imaginary part of the amplitude, **c** temporal evolution of $|U|^2$, **d** contour plot of $|U|^2$ as a function of the propagation distance along the fiber in the scenario 3 ($\alpha_2 = 0$ and $\alpha_3 = 1$)

The profiles studied are represented in relation to the normalized temporal variable t and the spatial coordinate z. The temporal transformation method used directly maps the temporal evolution of the electric field and does not require the use of the slowly varying envelope approximation. All input pulses studied exhibit symmetry with

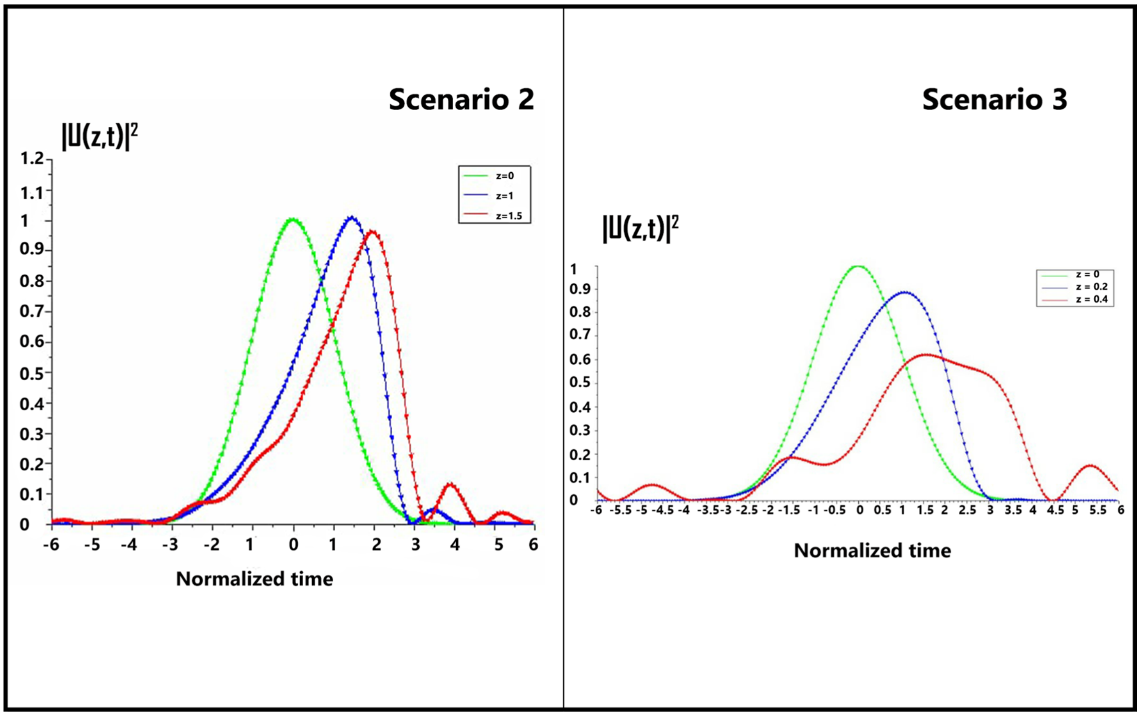

Fig. 4 The temporal evolution of the pulses at propagation distances selected according to scenarios 2 and 3

periodic boundary conditions, but upon propagation the symmetry is broken and the pulse becomes more asymmetric due to nonlinearities and third order dispersion. Indeed, under the influence of third-order dispersion, the shape and time spectrum of the pulse change in a complicated manner. According to Figs. 2 and 3, when propagating over a large distance, the oscillation of the envelope is greater, leading to a broadening of the temporal spectrum into several peaks, as pointed out by Van Cao et al [14]. The high-order nonlinearity effect (anti-cubic contribution) leads to the formation of a steep front on the trailing edge of the pulse. The pulse becomes more asymmetric in its propagation like [14]. As seen with respect to the input pulse, the electric field extended over a much wider temporal range and its central peak shifted considerably backwards. The calculated energy of the input and output pulses shows that energy is conserved. The different profiles represented show a certain qualitative similarity with those obtained by Edah et al [3].

Figure 4 presents the intensity profiles of the input function $U(t, z = 0) = U_0 \cosh(t/5) \exp(-t^2/4)$ at specific distance during propagation for scenarios 2 and 3. It appears that as the propagation distance increases, the shape of the pulse distorts considerably.

Conclusion

In this article, we have developed a finite difference scheme coupled with the Runge Kutta method to study the propagation dynamics of ultrafast pulses in optical fibers. As ultrashort pulses are widely used in optical telecommunications, there will be no shortage of interest in such pulses to improve transmissions. The results obtained in this article with the develop method confirm the impact of higher order dispersive effects and higher-order nonlinear effect on the propagation of ultrashort pulses. The studied ultrashort pulse changes shape and the higher order dispersion distorts the pulse in such a way that it becomes asymmetric with an oscillatory structure close to its fronds.

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