



RESEARCH ARTICLE

LINEAR VISCOELASTIC MODEL OF BORASSUS WOOD: RHEOLOGICAL PARAMETERS

Robert Michozounnou^{1,2}, Valéry K. Doko^{1,2}, Agapi K. Houanou¹, Gérard L. Gbaguidi Aïsse^{1,2}, Amos E. Foudjet³ and Antoine Vianou¹

1. Laboratoire d'Energétique Et De Mécanique Appliquée (LEMA) - EPAC - Bénin.
2. Laboratoire De Matériaux Et Structures (LAMS) - ESGC Verechaguine A.K. - Bénin.
3. CRESA Forêt - Bois, Université De Dschang, Cameroun.

Manuscript Info

Manuscript History

Received: 10 November 2021

Final Accepted: 14 December 2021

Published: January 2022

Key words:-

Borassus, Creep, Viscoelastic modulus, Rheological parameters

Abstract

The aim of the present work is to study the viscoelastic model of Borassus Aethiopum Mart which is relevant for the characterisation of a Borassus Pre-stressed Concrete beam. Mechanical tests were carried out; mainly the creep test on typical Borassus Aethiopum Mart specimens from the Pahou-Ahazon forest gallery in southern Benin, Africa, subjected to a constant bending stress along the beam. The moisture content of the specimens is set at 12% and is obtained by kilndrying at ATC du Bois. The loading level of the specimens is set at 20% of the failure load. A methodology starting from the determination of the compressive and bending stresses of Borassus Aethiopum Mart for the identification of the optimal values of the dynamic elastic modulus E and the dynamic viscosity constant η of Borassus Aethiopum Mart with the non-linear least squares method applied to the Kelvin Voigt rheological model. The instantaneous modulus of elasticity E_0 was determined from Hooke's law. A series of validation tests were performed on the numerical model. Finally, a comparison of the numerical model with the rheological representation confirmed the previous results.

Copy Right, IJAR, 2022., All rights reserved.

Introduction:-

In the field of construction, wood is the material whose use is most beneficial to the environment because its production requires less energy than other materials. It is one of the natural products whose use is essential for the environment. It is used in many ways to meet the different needs of society. It is a renewable and biodegradable material and the process of obtaining it is less time-consuming than that of some other materials. The sustainable use of wood allows the storage of excess carbon in the atmosphere. Wood is also a material that can be used structurally, especially in the construction field. It is a natural material whose physical and mechanical characteristics can be significantly influenced by the level of stress and duration of loading. Therefore, a good knowledge of its physical and mechanical properties is essential. However, studies on the instantaneous and delayed behaviour of wood are less extensive, so that the information is mostly superficial. It is necessary to test it under different types of loading. According to some previous studies on wood, several aspects are not taken into account from purely elastic modelling even in the case of low loads. This being the case, several investigations have been undertaken and have led to the conclusion that the deferred behaviour of wood is linear viscoelastic for stresses in the range [0% ; 35%] of the failure stress. Thus the deferred behaviour of wood can be represented by a

Corresponding Author:- Robert Michozounnou

Address:- Laboratoire d'Energétique Et De Mécanique Appliquée (LEMA) - EPAC - Bénin.

Kelvin Voigt model. The aim of this study is therefore to investigate the viscoelastic model of *Borassus Aethiopum* Mart (roastwood) from Benin.

Material and Methods:-

Material:-

The equipment used for the creep test is mainly composed of a bracket and a suspension plate. This equipment is identical to that proposed by Professor A. FOUDJET in his work in 1986.

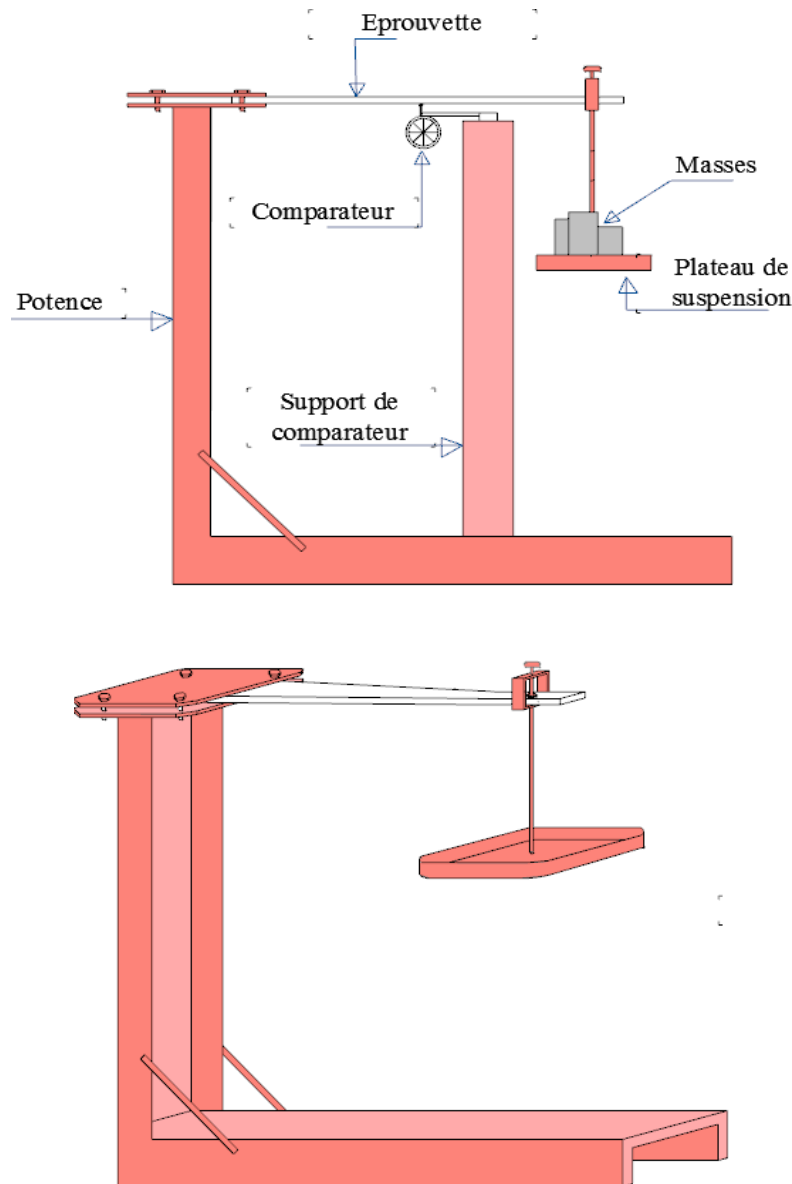


Figure 1:- Creep test material.

Plant material

The specimens designed for the creep tests were taken from the outer crown of *Borassus* logs in the longitudinal direction. Thus, nine (09) specimens of iso stress form were collected according to the configuration in Figure 2. For our study, the specimens have a moisture content of 12% obtained by drying in the ATC du Bois company in Allada (Benin) and are carefully wrapped in aluminium foil. During the entire test period, which lasted 15 hours, the specimens were weighed immediately. The test specimens were weighed immediately before and after the test. To ensure a smooth operation, we adopted the practical form presented below.

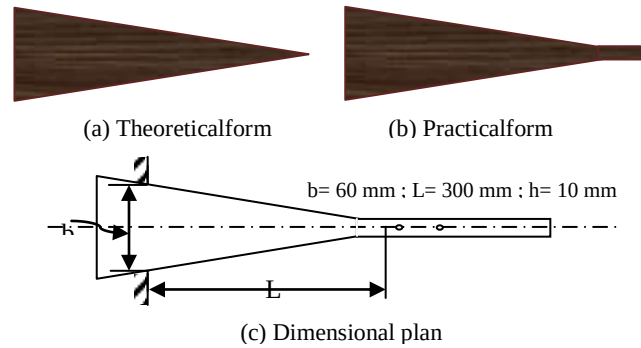


Figure 2:-Iso stress specimen of two-point creep test.

Methods:-

Principle of the test

The test consists of subjecting Borassus specimens to two-point bending with a uniform maximum stress on the fibres. During the test, a concentrated load is applied to the free end of the specimen. The extreme fibre stress is then maintained at 20% of the bending failure stress, i.e. 18.90 MPa. The deflection is read every 30 minutes at a distance of 150 mm from the end on which the load is displaced, by means of a dial indicator, whose resolution and travel are 0.01 mm and 10 mm respectively.

Kelvin-Voigt model

The model consists of a series of Kelvin-Voigt models and a spring. The spring characterises the instantaneous deformation, hence the instantaneous elastic modulus E_0 ; and the Kelvin-Voigt model characterises the creep, hence the dynamic elastic modulus E and the constant dynamic viscosity η .

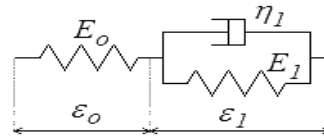


Figure 3:- Kelvin-Voigt model.

We will therefore rewrite with the natural notations: $\epsilon = \epsilon_0 + \epsilon_1$; $\sigma_0 = \epsilon_0 E_0 = \epsilon_1 E + \eta \frac{d\epsilon_1}{dt}$ (1)

Method for determining the deformation ϵ

The expression for the longitudinal deformation of the beam is established from the geometry of the deformed beam and is described as follows: $\epsilon = -\frac{2fy}{L^2 + f^2}$ (2)

Where: ϵ : beam deflection, L : beam length, y : distance from the tension fibre to the neutral axis.

The position of the neutral axis with respect to the tension fibre is determined using the trapezoidal normal stress distribution model on a complete surface proposed by PRAGER.

Based on the Navier-Bernoulli assumption and assuming that the squared moment of the beam section is constant.

The following relationship is established: $\frac{y}{h} = \frac{2\sigma_{uc}\sigma_{ut}}{(\sigma_{uc} + \sigma_{ut})^2}$ (3)

Where σ_{ut} is the tensile stress; σ_{uc} is the compressive stress; h is the height of the beam; y is the distance from the neutral axis of a tensioned fibre.

With equations (2) and (3) we have: $\epsilon = 4h \frac{f}{f^2 + L^2} \frac{\sigma_{ut}\sigma_{uc}}{(\sigma_{ut} + \sigma_{uc})^2}$ (4)

The method of identifying parameters

Using the non-linear least squares method and based on equation (5), the parameters η and E of the linear viscoelastic behavior law : $F(t, a, b) = \frac{\sigma_0}{a} \left(1 - e^{-\frac{a}{b}t}\right)$ Aveca = E , $b = \eta t \geq 0$ (5)

We adjust the measured deformations in order to minimize the distances d_i as follows: $d_i = \sum_{i=1}^n [y_i - F(t, a, b)]^2$ (6)

By means of equation (6) we have the iterative formulae of equation (7) below:

$$\begin{cases} a_{j+1} = a_j + \frac{(\sum_{i=1}^n A y_i - \sum_{i=1}^n A C) \sum_{i=1}^n B^2 - \sum_{i=1}^n A B (\sum_{i=1}^n B y_i - \sum_{i=1}^n B C)}{\sum_{i=1}^n A^2 \sum_{i=1}^n B^2 - (\sum_{i=1}^n A B)^2} \\ b_{j+1} = b_j + \frac{\sum_{i=1}^n A^2 (\sum_{i=1}^n B y_i - \sum_{i=1}^n B C) - (\sum_{i=1}^n A y_i - \sum_{i=1}^n A C) \sum_{i=1}^n A B}{\sum_{i=1}^n A^2 \sum_{i=1}^n B^2 - (\sum_{i=1}^n A B)^2} \end{cases}$$

With :

$$A = \frac{\partial}{\partial a} F(t_i, a_0, b_0) = \left[-\frac{1}{a_0^2} \left(1 - e^{-\frac{a_0}{b_0} t_i}\right) + \frac{t_i}{a_0 b_0} e^{-\frac{a_0}{b_0} t_i} \right] \sigma_0$$

$$B = \frac{\partial}{\partial b} F(t_i, a_0, b_0) = \left[-\frac{t_i}{b_0^2} e^{-\frac{a_0}{b_0} t_i} \right] \sigma_0$$

$$C = F(t_i, a_0, b_0) = \left[-\frac{1}{a_0} \left(1 - e^{-\frac{a_0}{b_0} t_i}\right) \right] \sigma_0$$

With equation (7), and according to the iteration stopping criterion defined for each parameter, the optimal values a and b were determined.

Iteration stopping criterion : $\frac{a_{j+1} - a_j}{a_j} < 10^{-5}$ et $\frac{b_{j+1} - b_j}{b_j} < 10^{-5}$

Results And Discussion:-

Table 1:- Key parameters of the creep test.

N°	Key parameters	Experimental values	Numerical values
1	Moisture content (%)	12	12
2	Initial strain ϵ_0 (%)	0,324	0,319
3	Final strain ϵ_f (%)	0,341	0,348
4	Creep coefficient K	1,05	1,09
5	True creep (%)	0,017	0,029
6	Relative creep	0,052	0,091
7	Request rate (%)	20	20
8	Instantaneous elastic modulus E_0 (MPa)	5835	5935

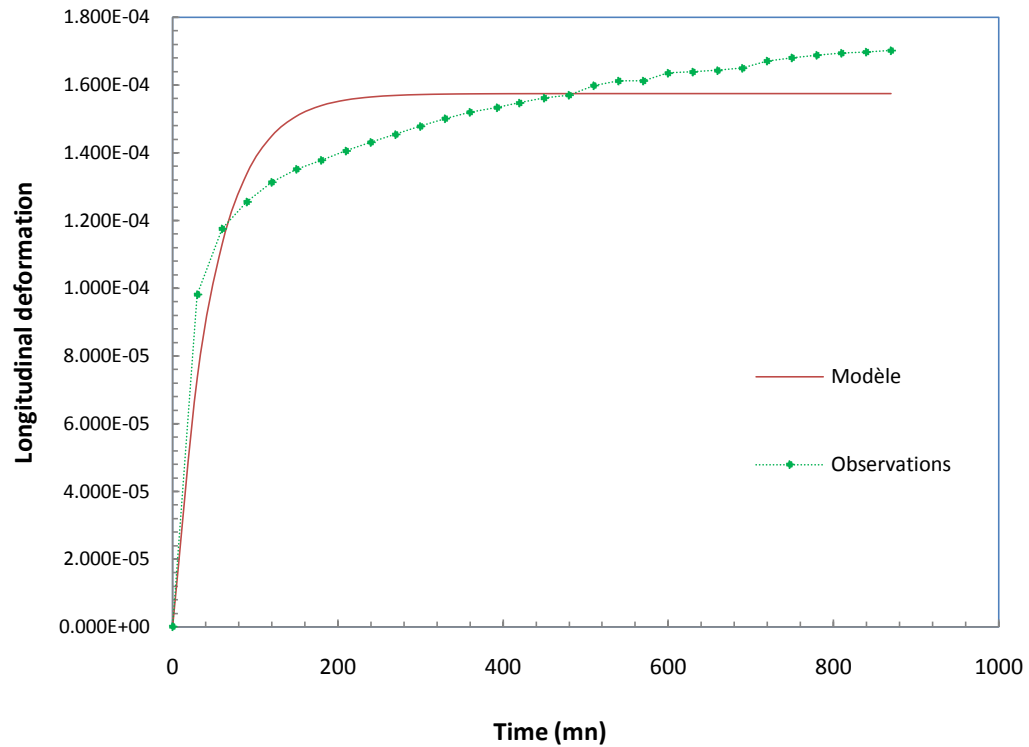


Figure 4:- Longitudinal deformation curves of the Borassus over time.

In view of the results obtained, Table 1 presents the main characteristics of the creep test, such as the initial deformation ε_0 , the final deformation f , the creep coefficient K , the natural creep $\Delta\varepsilon$ and the instantaneous modulus E_0 . The different studies are used to model the linear viscoelastic behaviour of wood and to validate the model. Figure 4 shows the experimental and simulated curves of the studied Borassus wood with respect to the evolution of the longitudinal deformation.

Table 2:- Optimal values of the rheological parameters of the model.

N°	Rheological parameters		Optimal values
1	Dynamic modulus of elasticity E	MPa	120 000
2	Dynamic viscosity constant η	MPa.mn	5 700 000

The optimal values of the rheological parameters (E and η) of the model obtained with a precision of 10^{-7} by minimizing equation (6) and using Excel software were presented in Table 2.

Tables 3 and 4 below present the statistical values of the tests for each parameter as well as the different t-distribution tests carried out using the t-distribution law and considering a 1% risk; in order to confirm the values obtained. It should be noted that the level of precision of the results does not require the determination of the confidence interval of the values.

Table 3:- Statistical values.

Indicators			
Nomenclature	Formula	Description	Result
S_{xx}	$\sum_{i=1}^n (x_i - \bar{x})^2$	Sum of squares of x	2,022E+06

SS_E	$\sum_{i=1}^n (y_i - \hat{y}_i)^2$	Errorsum of squares	8,71058E-12
SS_R	$\sum_{i=1}^n (\hat{y}_i - \bar{y})^2$	Regressionsum of squares	8,62718E-11
SS_T	$\sum_{i=1}^n (y_i - \bar{y})^2$	Total correctedsum of squares	8,62389E-11
$\hat{\sigma}^2$	$\frac{SS_E}{n-2}$	Unbiaisedestimator	3,11092E-13
$se(\hat{\beta}_2)$	$\sqrt{\frac{\hat{\sigma}^2}{S_{xx}}}$	Estimated standard error of the slope	3,92194E-10
$se(\hat{\beta}_1)$	$\sqrt{\hat{\sigma}^2 \left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}} \right)}$	Estimated standard error of the intercept	1,98898E-07
$\hat{\beta}_1$	E	Estimator de E	12E+04
$\hat{\beta}_2$	H	Estimator de η	5,7E+06

The number n of observations is equal to 30.

Table 4:- Statistical validation of parameters $\hat{\beta}_1$ and $\hat{\beta}_2$

Type of parameter	Statistical test	Data
parameter $\hat{\beta}_2$	Hypotheses	$H_0: \hat{\beta}_2 = 0$; $H_1: \hat{\beta}_2 \neq 0$
	$t_0 = \frac{\hat{\beta}_2}{se(\hat{\beta}_2)}$	1,4534E+16
	For $\alpha=1\%$ and using a t-distribution, we have : $t_{\alpha/2, (n-2)}$	2,763
	Conclusion	$t_0 > t_{\alpha/2, (n-2)}$ therefore we reject H_0 . $\hat{\beta}_2 = 5,700E+06$
parameter $\hat{\beta}_1$	Hypotheses	$H_0: \hat{\beta}_1 = 0$ $H_1: \hat{\beta}_1 \neq 0$
	$t_0 = \frac{\hat{\beta}_1}{se(\hat{\beta}_1)}$	6,0332E+11
	For $\alpha=1\%$ and using a t-distribution, we have : $t_{\alpha/2, (n-2)}$	2,763
	Conclusion	$t_0 > t_{\alpha/2, (n-2)}$ therefore we reject H_0 . Hence $\hat{\beta}_1 = 120\ 000$

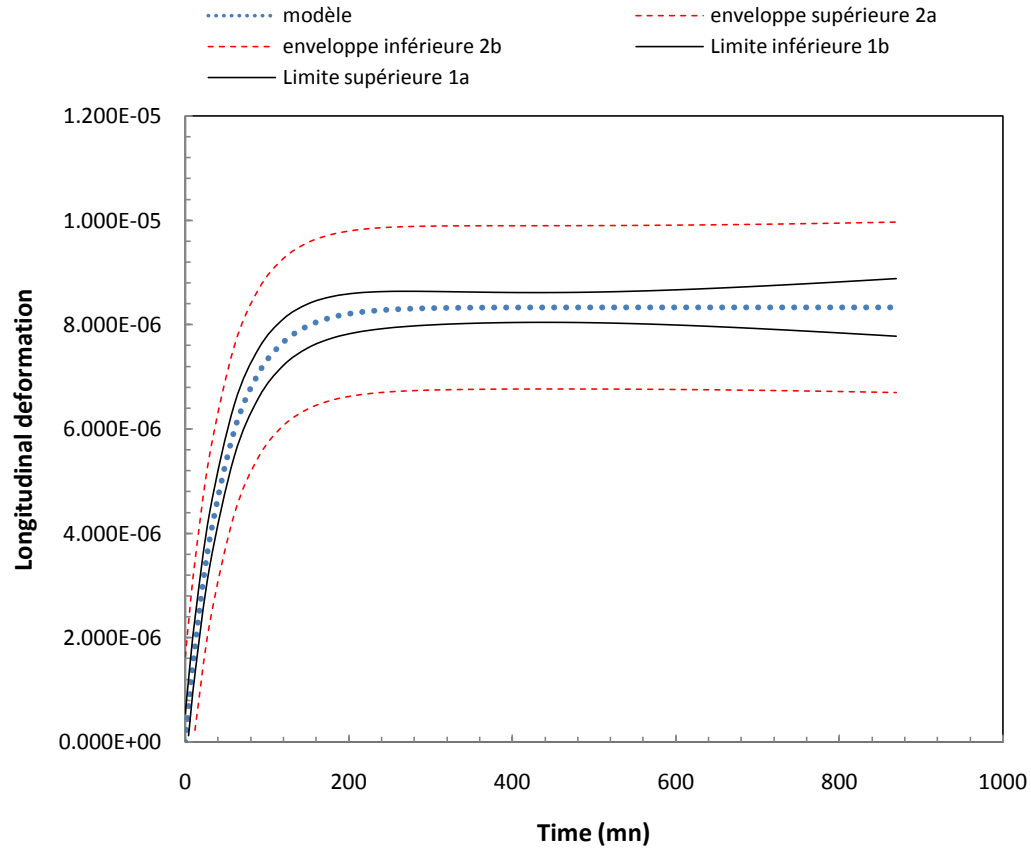


Figure 5:- Envelope curves of the linear viscoelastic model prediction.

Table 5:- Boundary values of the rheological parameters of the prediction fields.

Curves	Parameters	
	E^*	η^*
1a	112 400	5 700E+03
1b	133 200	5 700E+03
2a	105 840	5 700E+03
2b	148 500	5 700E+03

Figure 5 shows the prediction field boundaries of the linear viscoelastic model which are plotted using the statistical law. These are the 99% confidence interval on the solid line regression and the 99% prediction interval for a dashed line future observation. Table 5 summarises all the rheological parameters characterizing each limit.

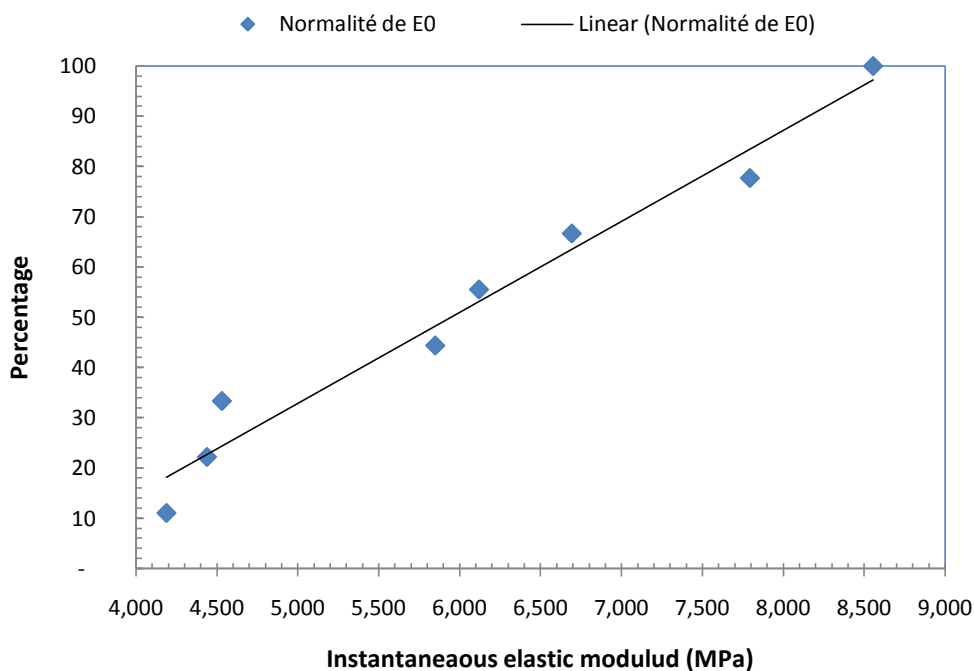


Figure 6:- Normalitycurve of the instantaneousmodulus of elasticity.

Figure 6 shows the normalitycurve of the instantaneouselasticmodulus. This probabilitydiagram of the distribution of the instantaneouselasticmodulus values calculatedusingHOOKE'slaw shows that the values are normallydistributed and validates the instantaneous elasticmoduluspresented in Table 1.

Table 6:- Coefficient of determination R^2

Statistical test			Data
Coefficient determination R^2	of	Formula	$R^2 = 1 - \frac{SS_E}{SS_T}$
		Value	
Interpretation			The model helpsaccount for 89,9% of the explained values

Table 7:- Values of statisticalindicators.

Indicators			Data
Notation	Expression	Designation	Calculated values
S	$\sum_{i=1}^n E_{0i}$	Sum	56715,573
$\bar{E}_0$	$\frac{S}{n}$	Samplemean	6301,730303
S_{xx}	$\sum_{i=1}^n (E_{0i} - \bar{E}_0)^2$	Sum of the squares of sampledeviation	2022497
S^2	$\frac{S_{xx}}{n-1}$	Sample variance	252812,125
S	$\sqrt{S^2}$	Sample standard deviation	502,804

The number of observations n isequal to 9.

Table 8:- Validation of E_0 and values of the lower and upperbounds of the confidence and predictioninterval.

	Statistical values	Data
VALIDATION OF E_0	Hypotheses	$H_0 : E_0 = \mu$ $H_1 : E_0 > \mu, \mu = 5835$
	$t_0 = \frac{E_0 - \mu}{S/\sqrt{n}}$	2,785
	For $\alpha=1\%$ and using t-distribution, we have : $t_{\alpha/2,(n-1)} = 3,106$	
	Conclusion	$t_0 < t_{\alpha/2,(n-1)}$ therefore we accept H_0 . Hence $E_0 = 5835$
CONFIDENCE INTERVAL	For $\alpha=1\%$ and using t-distribution, we have : $t_{\alpha/2,(n-1)}$	
	$\Delta E = t_{\alpha/2,(n-1)} \cdot \frac{S}{\sqrt{n}}$	521
	E_0	5835
	Lowerbound	5314
	Upperbound	6356
PREDICTION INTERVAL	For $\alpha=1\%$ and using t-distribution, we have : $t_{\alpha/2,(n-1)}$	
	$\Delta E = t_{\alpha/2,(n-1)} \cdot S \sqrt{1 + \frac{1}{n}}$	1646
	E_0	5835
	Lowerbound	4189
	Upperbound	7481

Table 6 presents the first test of the model's fit by calculating the coefficient of determination for Borassus Aethiopicum Mart. According to the results, the model accounts for 89.90% of the variability in the data.

Tables 7 and 8 present the values of the statistical indicators on the determination of the mean, the confidence interval and a prediction of future observation. The validation of the mean value of the instantaneous elastic modulus was based on the t-distribution law with a 1% risk for verification. The confidence and prediction intervals were also validated using the t-distribution table with a 1% risk of verification.

Conclusion:-

This study carried out on Borassus wood through two-point bending creep tests made it possible to determine the rheological parameters of the linear viscoelastic behaviour of the latter. The model used in this study was that of Kelvin Voigt, and the characteristics of the basic bricks, namely the spring and the damper, were determined, reflecting the instantaneous and delayed behaviour of the material respectively. From this work, some key indicators of Borassus wood such as the creep coefficient, the instantaneous modulus of elasticity etc. were also found. This study of the deferred behaviour of wood will therefore be necessary for the prediction of future deformations of a structural element made of Borassus wood. Finally, the results showed that the free flow of Borassus wood can be characterised in the evaluation of the linear viscoelastic behaviour by adopting the Kelvin-Voigt rheological model with the bending creep test when performed for a maximum loading time of 15 hours.

References:-

1. Trouy-Triboulot M.C. and Triboulot P., Matériau bois, structure et caractéristiques, Technique de l'Ingénieur, traité de construction, C925, 26 (2009)
2. Foudjet A., Contribution à l'étude rhéologique du matériau bois. Thèse ès Science en Génie Civil, Université Claude Bernard de Lyon (1986)
3. Bourahla N., Résistance des matériaux de base, GECOTEC, Alger (2003)
4. Bousaid E., Contribution à la modélisation des effets différés du bois et du béton sous conditions climatiques variables, Application aux structures mixtes bois-béton, Thèse en Structures, INSA de Lyon (2003)

5. Chevalier Y., Comportement élastique et viscoélastique des composites. Technique de l'Ingénieur, traité de Plastiques et composites, A 7 750, 32 (2009)
6. Houanou K. A, Tchéhouali A, Foudjet A., Identification of rheological parameters of the linear viscoelastic model of species of tropical woods (2012)
7. Husson J.M., Loi de comportement viscoélastique avec effet mémoire. Application à la mécanosorption dans le bois, Thèse en Génie Civil. Université de Limoges (2009)
8. Montgomery D.C., Runger G.C., Applied Statistics and Probability for Engineers, Third edition, John Wiley and Sons, Inc. Arizona State University (2003)
9. Nocedal J. and Wright S.J., Numerical Optimization, Springer, New York (1999)
10. Salençon J., Viscoélasticité pour le calcul des structures. Les Editions de l'Ecole Polytechnique, Les Presses des Ponts et Chaussées, 156 (2009).