Time-frequency Approach to Gaussian and Sinh-Gaussian Pulse Profiles Propagating in a Dispersive Medium

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In this paper, we use time-frequency analysis for pulse propagation in a linear dispersive medium. The Gaussian and sinh-Gaussian profiles are used as input ultra-short pulses $E(0,t)$ in the time domain. $\widetilde{E}(0, \omega)$ is the corresponding spectral profile. This spectral profile is decomposed into several wavelets centered at frequency Ω and the propagation constant $\beta(\omega)$ is expanded to the third order in Taylor series. Using inverse Fourier transform, we obtain numerically the profile of the pulse simultaneously both in time and frequency by means of Matlab software. We investigate the influence of the third order dispersion coefficient. Khelladi et al. previously used a Gaussian input pulse to derive time-frequency profile of a pulse up to second order. Our work is an extension up to third order by using Gaussian input pulse and sinh-Gaussian input pulse. The result showed that the presence of the third order dispersion term exhibits broadening of the pulse, and the sinh-Gaussian pulse undergoes broadening at much slower rate in comparison to Gaussian pulse.

1. Introduction

Progress in ultrafast optics relies extensively on the development of ways to characterize and manipulate dispersion. The growing number of femto-second lasers in industry, medicine and communication sector has increased the need for measuring the dispersive properties of media beyond that of glass and quartz [1]. Because of their broad bandwidth, femto-second lasers are particularly sensitive to chromatic dispersion characteristics of materials, in particular second-order (k'') and their third order dispersion, (k'''), which typically cause pulse broadening [2]. The study of propagation of ultrashort pulse in dispersive or complex media at the higher order dispersion terms is attracting growing interest, both in theory and experiment, in particular the effect of the third order dispersion. In [3], a general expression for the pulse shape is derived analytically and an asymptotic approximation when the third order dispersion is small. The general form of the pulse shape up to second order dispersion is analytically derived and the effect of third-order dispersion is discussed. In [4], the second-order dispersion is the main factor limiting transmission length, it reduces significantly the amplitude and broadens the time width of the pulses and the third order dispersion limits the transmission distance. In [5] and [6], Cojocaru derived some analytical dispersion expressions and concluded that both second and third-order dispersions may be compensated if one keeps the higher order ones as low as possible. The majority of the published articles (see [13] and references therein) deal with either frequency-domain or time-domain. As distinct from the Fourier transformation employed traditionally for analysis of signals, time-frequency analysis provides two-dimensional sweep of the studied one dimensional signal. Wavelet analysis is a kind of mathematical microscope, which ensures a good temporal resolution at different time-scales of a one-dimensional signal. The temporal shape of an ultrashort laser pulse may change upon propagating through a linear dispersive medium having a phase shift $\Phi(\omega)$. The change can be characterized by the coefficients of a Taylor series of the phase shift, which are calculated around the central frequency.

This work is based on a recent result by [7] on the time-frequency decomposition of an ultrashort pulse propagating through a linear medium and wavelet analysis of extremely short femto-second pulse propagating in dispersion-shifted single-mode fiber [8]. Following the methodology developed by them, our work is the extension up to the third order dispersion presented in [7].

The paper is organized as follows. Sec. 2 is devoted to plane wave analysis and dispersion relations. Sec. 3 deals with the time-frequency analysis for both profiles, and Sec. 4 contains the results and the discussion.

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2. Plane Wave Analysis and Dispersive Relations

A light source modulated by a Gaussian pulse propagating in a medium along the z-direction with angular frequency ω_0 can be described by

$$E(t) = E_0 exp(-\Gamma t^2) cos(\omega_o t)$$

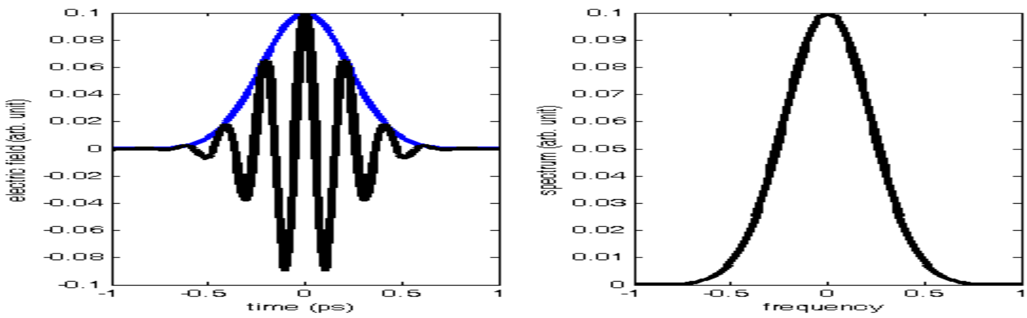

Fig.1: On the left, electric field (lower) and its intensity (upper); on the right, its spectrum

For signals characterized by a slow temporal varying envelope, the phase is usually approximated by a Taylor expansion in the neighborhood of the central frequency of the input pulse. The spectral amplitude $\tilde{E}(\omega, z)$ changes with z as [11]

$$\tilde{E}(\omega, z) = \left[\tilde{E}_0(\omega) e^{j\theta(\omega)}\right] exp\left[j \int_0^z \beta(\omega, z) dz\right] \quad (1)$$

The pulse response $E(t, z)$ at a point M of coordinate z in the medium

$$\tilde{E}(t, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{E}(\omega) \exp[j(\omega t - \beta(\omega) z)]\, d\omega$$

In order to understand a pulse shape further, the spectral phase of a femto-second laser pulse can be expressed through the use of a Taylor series about the carrier frequency ω_0 as follows

$$\Phi(\omega) = \Phi(\omega_0) + (\omega - \omega_0) \frac{d\Phi}{d\omega} + \frac{1}{2}(\omega - \omega_0)^2 \frac{d^2\Phi}{d\omega^2} + \frac{1}{6}(\omega - \omega_0)^3 \frac{d^3\Phi}{d\omega^3} + \cdots \quad (2)$$

Where, the derivatives are evaluated at ω_0 and the terms (left to right) describe the phase at the carrier frequency ω_0, the group delay, the group-delay dispersion (GDD), the third-order dispersion (TOD), respectively. It is worth mentioning that the group-velocity dispersion (GVD) is related to the second derivative of refractive index with respect to wavelength, where the terms corresponding to the different orders have the following meaning: the zero-order term describes a common phase shift. The first-order term contains the inverse group velocity (i.e., the group delay per unit length) and describes an overall time delay without an effect on the pulse shape. The second-order (quadratic) term contains the second-order dispersion (SOD) or group delay dispersion (GDD) per unit length. The third-order (cubic) term contains the third-order dispersion (TOD) per unit length. The dispersion of various orders for a medium can most conveniently be calculated if the refractive index is specified with a kind of Sellmeier formula [9]. Using Eqn. (2) up to second order term, we get the following result

$$E(t, z) = \frac{E_0}{\pi} \sqrt{\frac{\Gamma(z)}{2}} exp\left[j\omega_0 \left(-\frac{z}{V_g(\omega_0)}\right)\right] exp\left[-\Gamma(z) \left(-\frac{z}{V_g(\omega_0)}\right)^2\right] \quad (3)$$

Where, $V_\phi(\omega_0) = \left(\frac{\omega}{k}\right)_{\omega=\omega_0}$; $V_g(\omega_0) = \left(\frac{d\omega}{dk}\right)_{\omega=\omega_0}$ and $\frac{1}{\Gamma(z)} = \frac{1}{\Gamma} + 2jkz$

$$k''(\omega_0) = \left(\frac{d^2k}{d\omega^2}\right)_{\omega=\omega_0} = \frac{d}{d\omega}\left(\frac{d\omega}{dk}\right)_{\omega=\omega_0}$$

The term $k''(\omega_0) = \left(\frac{d^2k}{d\omega^2}\right)_{\omega=\omega_0}$ is called the group velocity dispersion or GVD and is defined as the propagation of different frequency components at different speeds through a dispersive medium. This is due to the wavelength-dependent index of refraction of the dispersive material. Because the GVD is frequency-dependant, higher order dispersive effects can also play a role in the propagation of the pulse

$$k''' = -\frac{\lambda^2}{4\pi^2 c^3}\frac{d}{d\lambda}\left(\lambda^3 \frac{d^2 n}{d\lambda^2}\right) \quad (4)$$

The introduction of the third order dispersion characterized by β_3 permits to introduce the next important parameter S defined by $S = \frac{dD}{d\lambda} = \left(\frac{2\pi c}{\lambda}\right)^2 \beta_3 + \left(\frac{4\pi c}{\lambda^3}\right)\beta_2$. Various terms corresponding to the different orders have the following meaning: the zero-order describes a common phase shift; the first-order term contains the inverse group velocity (group delay per unit length) and describes an overall time without an effect on the pulse shape; the second-order (quadratic) term contains the second order dispersion or group delay dispersion per unit length; the third order (cubic) term contains the third order dispersion per unit length. Here, n is the refractive index of the core, c is the velocity of light in vacuum and λ is the wavelength expressed in micron. For the refractive index n, we used the Sellmeier formula [9]

$$n^2 = 1 + \sum_{m=1}^{3} \frac{a_m \lambda^2}{\lambda^2 - b_m} \quad (5)$$

Where,

$a_1 = 0.6961663 \qquad b_1 = 0.004679148$
$a_2 = 0.4079426 \qquad b_2 = 0.01351206$
$a_3 = 0.8974994 \qquad b_3 = 97.934002$

Including the TOD term, $E(t,z)$ is evaluated numerically.

3. Time-frequency Analysis

The electric field associated to the wavelet centered at Ω, for both Gaussian and hyperbolic sine Gaussian-pulses can be evaluated by

$$\tilde{T}(\omega,\Omega) = \tilde{E}(\omega) exp\left[-\frac{(\omega-\Omega)^2}{4\gamma}\right] \quad (6)$$

Where, $\tilde{E}(\omega)$ denotes the spectrum of the pulse given by $\tilde{E}(\omega) = E_0 \frac{1}{2\sqrt{\pi\Gamma}} exp\left(\frac{-(\omega-\omega_0)^2}{4\Gamma}\right)$ (for Gaussian pulse). In the time domain, we used inverse Fourier transform, then

$$T(t,\Omega,z=0) = FT^{-1}[\tilde{T}(\omega,\Omega,z=0)]$$

As a result, after performing the Fourier transform, we have

$$T(t,\Omega,z=0) = E_0\sqrt{\frac{\gamma}{\Gamma+\gamma}}\, exp\left[-\frac{(\omega_0-\Omega)^2}{4(\gamma+\Gamma)}\right] exp\left[-\frac{\gamma\Gamma}{\gamma+\Gamma}t^2\right] exp\left[j\frac{(\gamma\omega_0+\Omega\Gamma)}{(\gamma+\Gamma)}\right] \quad (7)$$

We conclude that with respect to time the pulse is also Gaussian for parameter $\frac{\gamma\Gamma}{\gamma+\Gamma}$ showing that this parameter controls the width of the Gaussian function.

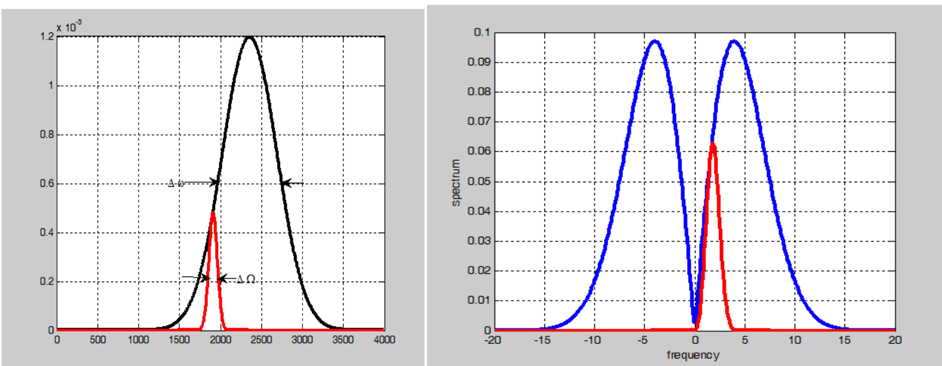

Fig.2: The left-hand side shows the wavelet inside Gaussian envelope centered at ω_0 whereas the right-hand side shows the wavelet inside Gaussian envelope centered at zero. Each spectrum is a set of several wavelets centered at Ω. The wavelet for sinh-Gaussian at zero is symmetric with respect to zero with one branch in the negative domain and the other in the positive domain.

We consider the case of a sinh-Gaussian pulse for which the incident field at $z = 0$ may be assumed have the following form [10]

$$q(0,t) = A_0 exp\left[-\frac{(1+iC_0)t^2}{2T_0^2}\right] sinh(\Lambda t)$$

Where, $\Lambda = \frac{\Omega_0}{T_0}$ and the field spectrum is obtained by

$$\widetilde{E_S}(0,\omega) = \frac{A_0}{2} T_0 \sqrt{\frac{2\pi}{1+iC_0}}\, exp\left(\frac{\Omega_0^2}{2}\right) exp\left(\frac{-T_0^2\omega^2}{2(1+iC_0)}\right) \left[exp\left(\frac{i\omega\Omega_0 T_0}{1+iC_0}\right) - exp\left(\frac{-i\omega\Omega_0 T_0}{1+iC_0}\right)\right] \quad (8)$$

The wavelet associated with the unchirped sinh-Gaussian pulse in the time domain is given by

$$\theta(t,\Omega,z=0) = T_0 \exp\left(\frac{\Omega_0^2}{2}\right) \exp(\mathrm{i}\Omega \mathrm{t}) \left[\exp(\Omega\beta_1) \exp\left(\frac{\beta_p^2}{4\alpha_1^2}\right) - \exp(\Omega\beta_1^*) \exp\left(\frac{\beta_{q^*}^2}{4\alpha_1^2}\right)\right] \quad (9)$$

with $\beta_p = \alpha_1 + it$; $\beta_q = \beta_1 + it$; where $\alpha_1 = \left(\frac{T_0^2}{2} + \frac{1}{4\gamma}\right)$; $\beta_1 = i\Omega_0 T_O - \Omega T_0^2$.

3.1. Second order term

When the pulse propagates to the z position, the wavelet $\tilde{T}(\omega,\Omega,z)$ is

$$\tilde{T}(\omega,\Omega,z) = \frac{E_0}{2\sqrt{\pi\gamma}} \tilde{E}(\omega) exp\left[-\frac{(\omega-\gamma)^2}{4\gamma}\right] exp[-j\Phi(\omega)] \quad (10)$$

We must add that the wavelet is characterized by a Gaussian envelope and the decomposition is valid when $\Delta\omega \gg \Delta\Omega$; $\Delta\Omega$ belonging to the spectrum of the pulse. This implies that the duration of this wavelet is large enough to ensure that the analyzing function has only non-negligible values over a spectral range lying in the neighborhood. Because we do not know the detailed form of $\Phi(\omega)$, we expand the phase in Taylor series around the central frequency Ω. We, therefore, need a large number of components $\Delta\Omega$ to reconstruct the spectrum (see Fig. 2). Then, the Taylor decomposition is given by

$$\Phi(\omega) = \Phi^{(0)}(\Omega) + (\omega-\Omega)\left(\frac{d\Phi}{d\omega}\right)_{\omega=\Omega} + \frac{(\omega-\Omega)^2}{2}\left(\frac{d^2\Phi}{d\omega^2}\right)_{\omega=\Omega} + \frac{(\omega-\Omega)^3}{6}\left(\frac{d^3\Phi}{d\omega^3}\right)_{\omega=\Omega} + \cdots + \cdots \quad (11)$$

Up to second order term of the spectral phase in the time domain the wavelet for the Gaussian pulse can be expressed by

$$T(t,\Omega,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{\mathrm{T}}(\omega,\Omega,\mathrm{z}) \exp[\mathrm{j}\omega\mathrm{t}]\mathrm{d}\omega$$

Performing this integration by setting

$$\frac{1}{\Gamma(z)} = \frac{1}{\Gamma} + \frac{1}{\gamma} + 2jk''(\Omega)$$

We have

$$T(t,\Omega,z) = \frac{E_0}{4\pi^2}\sqrt{\frac{\Gamma(z)}{\Gamma}}\, exp\left[-\frac{(\Omega-\omega_0)^2}{4\gamma}\left(1 - \frac{\Gamma(z)}{\Gamma}\right)\right] exp\left[-\Gamma(z)\left(t - \frac{z}{V_g(\Omega)}\right)\right] exp\left[j\left(\Omega t - \Phi^{(0)}\right)\right] \times exp\left[-j\left(t - \frac{z}{V_g(\Omega)}\right)\left(\frac{\Omega-\omega_0}{\Gamma}\right)\right] \quad (12)$$

As far as sinh-Gaussian pulse is concerned

$$T(t,\Omega,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \widetilde{E_S}(0,\omega)\, exp\left(-iz\left(\Phi(\Omega) + (\omega-\Omega)\left(\frac{d\Phi}{d\omega}\right)_{\omega=\Omega} + \frac{(\omega-\Omega)^2}{2}\left(\frac{d^2\Phi}{d\omega^2}\right)_{\omega=\Omega}\right)\right) exp[j\omega t]d\omega \quad (13)$$

3.2. Third order term

Using the Fourier decomposition up to the third order term, we should evaluate the following integral

$$T(t,\Omega,z) = \frac{1}{4\pi\sqrt{\pi\Gamma}}\int_{-\infty}^{\infty}\tilde{T}(\omega,\Omega,z)\,exp[j\omega t]d\omega$$

$$T(t,\Omega,z) = \frac{1}{4\pi\sqrt{\pi\Gamma}} \int_{-\infty}^{\infty} exp\left[\frac{(\omega-\omega_0)^2}{4\Gamma}\right] exp\left[\frac{(\omega-\Omega)^2}{4\gamma}\right] exp[j\omega t]\,d\omega \times exp\left[-j\left(\Phi^{(0)} + (\omega-\Omega)\Phi^{(1)} + \frac{1}{2}(\omega-\right.\right.$$
Ω2Φ2+16ω−Ω3Φ3

(14)

Let's denote: $a = \frac{1}{\Gamma} + \frac{1}{\gamma} + 2jk''(\Omega)$ and $b = \frac{\Omega-\omega_0}{2\Gamma} + j\beta_1 z$, Eqn. (14) can be rewritten in the following form

$$T(\omega,\Omega,z) = \frac{1}{2\sqrt{\pi\Gamma}} exp[-j\beta_0 z] exp\left[\frac{(\Omega-\omega_0)^2}{4\Gamma}\right] \times U \quad (15)$$

Where

$$U = \frac{1}{2\pi}\int_{-\infty}^{\infty} exp\left[-a(\Omega-\omega_0)^2 - b(\omega - \Omega) exp\left[-j\frac{1}{6}(\omega - \Omega)^3\beta_3 z\right] exp[j\omega t]d\omega\right]$$

Let's use the identity

$$exp\left[-a(\Omega-\omega_0)^2 - b(\omega - \Omega) exp\left[-j\frac{1}{6}(\omega-\Omega)^3\beta_3 z\right]\right. = exp(a\omega^2 - b\omega) exp\left[-j\frac{1}{6}(\omega)^3\beta_3 z\right] \otimes \delta(\omega - \Omega)\Big]$$

Then

$$U = FT^{-1}\left[exp(-a\omega^2 - b\omega) exp\left(-j\frac{1}{6}\omega^3\beta_3 z\right) \otimes \delta(\omega - \Omega)\right] = exp(j\Omega t) FT^{-1}\left[exp(-a\omega^2 - b\omega) exp\left(-j\frac{1}{6}\omega^3\beta_3 z\right)\right]$$

Considering $F(z,\omega) = exp(-a\omega^2 - b\omega)$ as the Fourier transform of a function $f(z;\,t)$ and $G(z;\,\omega) = exp\left(-j\frac{1}{6}\omega^3\beta_3 z\right)$ as the Fourier transform of function $g(z;\,t)$. The function $g(z;\,t)$ is evaluated by means of the Airy function defined as [11]

$$A_i(x) = \frac{1}{2\pi j}\int_{-j\infty}^{+j\infty} exp\left(xz - \frac{z^3}{3}\right)dz \quad (16)$$

Then, the convolution theorem is used to obtain

$$T(t,\Omega,z) = \frac{1}{2\sqrt{\pi\Gamma}} exp[j(\Omega t - \beta_0 z] exp\left[\frac{(\Omega-\omega_0)^2}{4\Gamma}\right] \int_{-\infty}^{\infty} f(u)g(u-t)du \quad (17)$$

The integral written in Eqn. (17) is evaluated numerically.

As far as sinh-Gaussian pulse is concerned,

$$T(t,\Omega,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\widetilde{E_S}(0,\omega)\,exp\left(-iz\left(\Phi(\Omega) + (\omega-\Omega)\left(\frac{d\Phi}{d\omega}\right)_{\omega=\Omega} + \frac{(\omega-\Omega)^2}{2}\left(\frac{d^2\Phi}{d\omega^2}\right)_{\omega=\Omega} + \frac{(\omega-\Omega)^3}{6}\left(\frac{d^3\Phi}{d\omega^3}\right)_{\omega=\Omega}\right)\right) exp[j\omega t]d\omega \quad (18)$$

4. Results and Discussion

Gaussian and hyperbolic sine-Gaussian profiles are used and the parameters used for simulations are

- Initial pulse: duration of 5fs;
- Wavelength: $\lambda = 800\,nm$;
- Pulse of the wavelet: duration of 1ps;
- Sinh parameter
- Ω_0: 0.5; 0.75; 1.0; 1.25; 1.50; 1.75; 2.0; 2.5; 3:
- Initial chirp: $C_0 = 0$

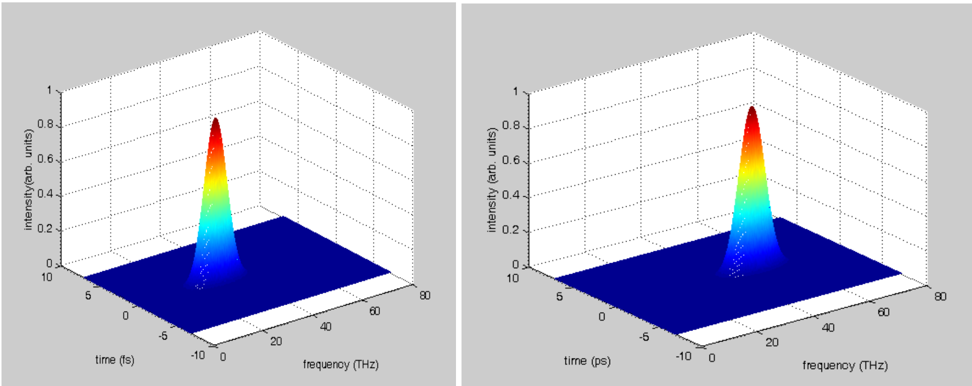

Fig.3: On the left-hand side, initial pulse, with $z = 0$. The wavelet is centered around Ω. This plot represents the electric field associated with the wavelet $T(\Omega, t, z = 0)$. On the right-hand side, pulse with second order dispersion after propagation of 1mm in the fused silica for Gaussian pulse.

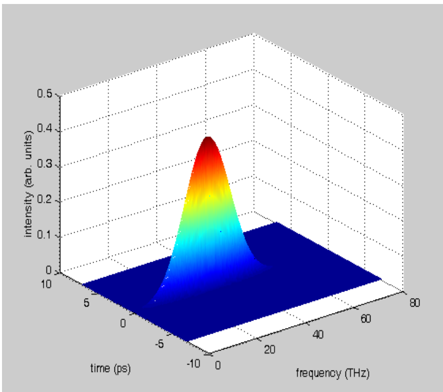

Fig.4: Pulse with third order dispersion included, after propagation of 1mm in the fused silica, for Gaussian pulse.

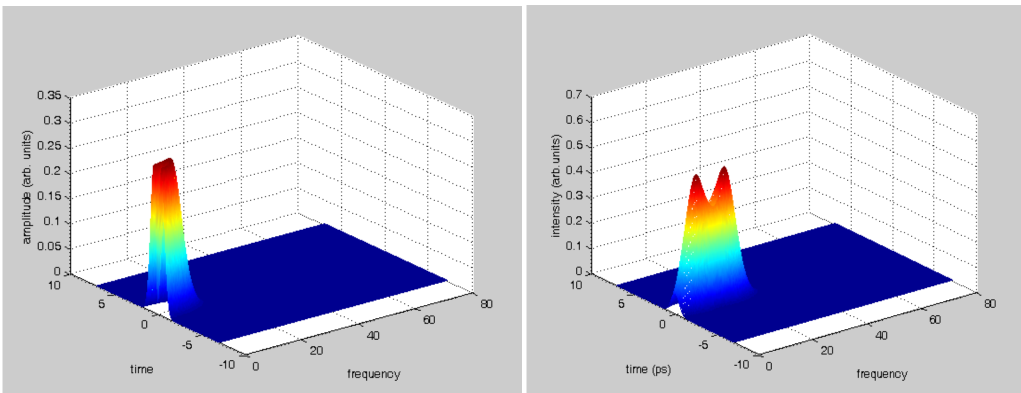

Fig.5. On the left, initial pulse, for sinh-Gaussian pulse for $\Omega_0 = 0.25$. On the right, pulse with third order dispersion included, after propagation of 2.3 mm in the fused silica, for sinh-Gaussian pulse, for $\Omega_0 = 0.25$.

Discussion

- Sinh-Gaussian is a superposition of two de-centered Gaussian pulses located at Ω_0 and $-\Omega_0$. For the same value of temporal parameter T_0 sinh-Gaussian pulses possess larger temporal width in comparison to Gaussian pulses.

- After propagation through the medium, the pulse is visualized in three dimensional representation, which permits the evaluation of the broadening or not of the pulse.

For Gaussian pulse

Let's point out that when the duration of the initial pulse increases, the width of the SOD and the TOD terms decreases. When the duration of the Gaussian pulse decreases, the widths of the SOD and TOD terms become large. So, the duration of the initial pulse has an effect on the pulse profile.

In Fig. 3 on the left, symmetry of the pulse is shown when propagation distance is zero. During propagation shown on the right, the pulses exhibited a distortion (asymmetry) as the distance increased. So, it is clear the pulse broadens significantly.

In Fig. 4, the broadening is also visible if the TOD is included. The shape of the pulse is distorted. Pulse distortion is caused by the fact that the group velocity is not constant with frequency. For the wavelets approach, the group delay dispersion and the third order dispersion are evaluated at: $\omega = \Omega$, which belongs to the spectrum of the electric field. The asymmetry of the pulse is caused by a strong increase of high order dispersion. Note that the wavelets analysis informs us about the variation of the instantaneous frequency; what infers the modulation of the electric field.

For sinh-Gaussian pulse

Fig. 5 on the left, shows initial pulse, for sinh-Gaussian profile for $\Omega_0 = 0.25$, whereas on the right, when third order dispersion is included, after propagation of 2.3 mm in the fused silica, for sinh-Gaussian pulse, for $\Omega_0 = 0.25$. An important point is that when the sinh parameter Ω_0 increases the broadening decreases. The presence of the third order dispersive term causes the broadening of the pulse. For the same width of both pulses, sinh-Gaussian pulse broadens at much slower rate as compared to Gaussian pulse.

5. Conclusions

In this paper, the equations for evaluating group velocity dispersion and TOD are derived. The temporal spreading was evaluated for 5 fs pulse duration and wavelength 800 nm, and the duration of the wavelet is 1ps. From the plot, GVD causes pulses spread in time as a result of different frequency components of the pulse traveling at different velocities that the initially unchirped pulse develops an added chirp in passing through the medium. This chirp depends on the material and on the initial duration of the pulse second order dispersion broadens the time width of the pulse, and the third order dispersion is also a broadening factor for pulse, and a limiting factor. It is important to take into account the third order dispersion term for ultrashort pulses and the pulse become larger and thus higher order dispersive terms can be omitted. For further work, we shall apply the method to resolve a problem in the nonlinear regime.

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