Constit Polit Econ (2007) 18:69–82
DOI 10.1007/s10602-006-9012-y

ORIGINAL ARTICLE

On the evaluation of power in parliaments and government formation

Nicolas G. Andjiga · Daoud Badirou · Boniface Mbih

Published online: 29 March 2007

Abstract This paper is devoted to the study of power in Parliaments. We explain how the power of coalitions can be computed after elections. We add to the existing literature by using this analysis to predict what government may emerge from these elections.

Keywords Power indices · Parliamentary method · Core · Coalition formation

JEL Classification C71

1 Introduction

Legislative elections are usually followed by the formation of a new government in most countries. This outcome is generally reached after a process of bargaining between different political parties, in order to form coalitions. The goal of this process is—at least for some groups—to implement a cooperation by which they can obtain a result that each of them prefers to what they could have obtained alone or within another coalition. Theorists have for a long time been interested in the analysis of the process of coalition formation. Taylor (1971) discusses a set of 'decision-control' theories, where it is assumed that each agent (each player) wishes to be a member of a winning coalition

N. G. Andjiga
Ecole Normale Supérieure, Université de Yaoundé, Yaoundé, Cameroon

D. Badirou · B. Mbih (✉)
Faculté de Sciences économiques, Université de Caen, UMR CNRS 6211, 14032 Caen, France
e-mail: boniface.mbih@unicaen.fr

(see Sect. 2) and to have as much control as possible over the decision of that coalition. To cite only two instances, Leiserson (1966) distinguishes a number of aims which motivate agents in the choice of their partners, such as the control of other agents, or to minimize ideological differences within the coalition; this same idea is developed by Axelrod (1970, p. 167), under the assumption that 'the less conflict of interest there is in a coalition, the more likely the coalition will form', which means that the most probable coalitions are those containing ideologically adjacent parties.

In this paper, we study the formation of a governmental coalition after general elections, using game-theoretic concepts and, more specifically, power indices. The literature on this subject is growing quickly; for an account on the main directions of the development of this topic, we refer the reader to Holler and Owen (2001). One important question is: given the weights of the agents in a game, how stable is a coalition once it is formed? We argue here that this question is crucial in the context of the formation of a government after general elections in democracies. Moreover, depending on the sociological environment, the coalitions that will form take account of both the ideological proximity of parties and the distribution of power inside the coalition. We then present two methods of prediction of a governmental stable majority in a democratic parliament, after the evaluation of the power of each party in all partitions, using Owen (1977) and Owen-Banzhaf (see Owen, 1981) notions of value. Our difference with previous research based on power indices (e.g. Owen & Carreras, 1988) is that this evaluation is done with two explicit alternative assumptions: (i) we first suppose that the party with the largest number of seats after the legislative elections forms the government: we call this the 'parliamentary method'; (ii) second, we apply the notion of the core to all the partitions. The choice of these two methods is based on their underlying notions of stability: in cooperative game theory, the concept of the core is usually associated with the idea of stability, and in real political life, the parliamentary method is considered as an effective way to reach stability. We illustrate these two methods by applying them to the parliaments of some countries, with the underlying games depending on the number of main parties—the number of players—in each country: on the one hand the Republic of Cameroon and the Czech Republic, where there were four main parties for the periods under study, and on the other hand the Republic of Germany and Catalonia (Spain) where there were five main parties. Catalonia was studied by Owen and Carreras (1988) and the structure of its parliament, given by the repartition of seats, is quite different from that of the German parliament (from the 2005 legislative elections), although both of them are five-game players. We thus compare these two games. Besides, the rationale for the choice of Cameroon is that unlike European democracies, in African countries and especially in Cameroon, political parties can by no means be ranked along an ideological axis; hence, this difference permits a comparison between two parliaments' games with and without an ideological axis, but with the same number of players. Finally we compare the results of the theory with real events.

The organisation of the paper is as follows: Section 2 introduces the basic notations and definitions; In Sect. 3 we explain the methods upon which our stability notions are based and; In Sect. 4 we apply them to 4-player games, and then to 5-player games. Section 5 then concludes the paper.

2 Notations and definitions

A *simple game* is a pair $G = (N, W)$, where N (the set of parties) is a non empty set and W is the nonempty set of all *winning coalitions* (or nonempty subsets of N). More intuitively, a simple game is a cooperative n-person game in which the set of all possible coalitions is partitioned into two subsets; on the one hand the subset of winning coalitions contains all groups that have the power to control the game and determine its outcome, and on the other hand all other coalitions—losing coalitions—have no control power. G is a *simple game with quota* if there exists a quota $q > 0$ and for each party i a non negative weight w_i such that S is winning if and only if the total weight of S, $w_s = \sum_{i \in S} wi$, is greater than or equal to q.

In this paper, a simple game with quota $G = [N, w_i, q]$ will be interpreted as formalizing the political configuration within a parliament, where N is the set of the political parties in the parliament, w_i is the number of deputies of the political party i, q is the number of deputies or the quota necessary to form a governmental majority at the parliament.

i is *pivotal* or *critical* for S in G if S is a winning coalition and S–$\{i\}$ is not a winning minimal coalition. The *marginal contribution* of i in S is 1 if i is critical for S in G and 0 otherwise. *S minimal* on G if S is a winning coalition that does not contain a winning (proper) coalition.

In what follows, we will be concerned with two power indices: Owen and Banzhaf values.

In Owen (1977) a modification of the Shapley value (1953) is suggested "to take into account the possibility that some players - because of personal or political affinities may be more likely to act together than others".

Let v be a simple game with $N = \{1, 2, 3, ..., n\}$ the set of players. An *a priori* union structure is a partition $T = [T_1, T_2, T_m]$ of N. The idea is that each T_j is an a priori union, i.e. a set of players who agree (in a more or less binding manner) to collaborate.

Letting $M = \{1, 2, 3, ...,m\}$, we define a quotient game $u = v/T$, with the set of players M, by $u(S) = v(\bigcup_{j \in S} T_j)$ for each $S \subset M$.

Let π be a permutation of N, and define the game $\pi\, v$ by $\pi\, v(K) = v\pi\,(K)$, with $K \subset N$ and $\pi\,(K) = \pi\, K$. Furthermore, let ρ be a permutation of M and let $T_{\rho_k} = \{j \in N : \rho^{-1}(j) \in T_k\}$

A *carrier* of v is any set $H \subseteq N$ with $v(S) = v(H \cap S)$ for all $S \subseteq N$

The modified value, φ, is a mapping which assigns an n-tuple $(\varphi_1, \varphi_2,...,\varphi_n)$ to each pair $[v,T]$ and which axiomatically is assumed to satisfy the following axioms:

A1. If H is a carrier for v, then $\sum_{i \in H} \varphi_i[v, T] = v(H)$.
A2. For any ρ, $\varphi[v; \{T_1, T_2, \ldots, T_m\}] = \varphi[v; \{T_{\rho 1}, T_{\rho 2}, \ldots, T_{\rho m}\}]$.
A3. For any $j \in M$, the quantity $\sum_{i \in H} \varphi_i[v, T]$ only depends on the quotient $u = v/T$.
A4. For any $\pi, \varphi_i[\pi v; \{T_1, T_2, \ldots, T_m\}] = \varphi_{\pi(i)}[v; \{\pi T_1, \pi T_2, \ldots, \pi T_m\}]$.
A5. For any two games, v and v' $\varphi[v, T] + \varphi[v', T] = \varphi[v + v', T]$.

Owen (1977) shows that there is a unique value satisfying these axioms, given by:

If $i \in T_j \in T$, then $\varphi_i[v, T] = \sum_S \sum_{K \subset T_j} \frac{k!(t_j - k - 1)!s!(m - s - 1)!}{t_j!m!}[v(Q \cup K \cup \{i\}) - v(Q \cup K)]$,

where $Q = \bigcup_{r \in S} T_r$ and k, s, t_j are the cardinalities of K, S, and T_j respectively.

Owen (1981) rewrites the Banzhaf value: "he takes into account the possibility that some players because of personal or political affinities—may be more likely to act together than others". Owen obtains a modification of the Banzhaf value:

If $i \in T_j \in T$, then $\varphi_i[v, T] = \sum_S \sum_{K \subset T_j} \frac{1}{2^{m + t_j - 2}}[v(Q \cup \{i\}) - v(Q \cup K)]$

where $Q = \bigcup_{r \in S} T_r$ and k, s, t_j are the cardinalities of K, S, and T_j respectively.

3 Two methods of government formation

We evaluate the power of every political party in these partitions, using the Owen (1977) and Owen-Banzhaf (see Owen 1981) notions of value. Considering successively each of these two values, our goal is to give a prediction of which government will be formed at the end of the game, on the basis of a list of criteria. We suggest the following alternative methods:

3.1 The parliamentary method

The party which obtains the largest number of seats is asked to form the government. For the sake of simplicity, we call that party 'party1'. Then, party 1 must choose the partition that satisfies the following criteria: the parliamentary method, and the method of the core.

Criterion 0: Party 1 chooses a partition which contains a winning coalition of which it is a member.

Criterion 1: Under the preceding constraint, party 1 must choose a partition in which it has the maximum power.

Criterion 2: The power of party 1 and its partners must be independent of the configuration of the parties of the opposition; in other words, the power of party1 and its partners must be the same, whether the Opposition is united or divided. We call this criterion the *external stability* condition.

Criterion 3: Party 1 must choose a partition which contains the smallest number of decisive parties: we call this the *internal stability* condition.

Criterion 4: criteria 2 and 3.

3.2 The core

The other method of forming a government that we suggest and analyse is based on the concept of the core.

After the evaluation of the power of every party in the partitions, we determine a set of winning coalitions, on which we define the dominance relation of a partition in the following way:

Let B_i and B_j be two partitions, we say that B_i dominates B_j if there exists a winning coalition in B_i such that each party in that coalition has an index power in B_i greater than its power index in B_j.

The core is the set of all undominated partitions.

4 Applications

In what follows below, we shall distinguish two categories of games according to the number of decisive players. We begin with four players; we shall then consider the five players case. Let us denote the partitions of $N = \{1,2,3,4\}$ as follows: $B_{(i,j,k,l)} = \{\{i\},\{j\},\{k\},\{l\}\}$ when every player is alone, $B_{(ijk,l)} = \{\{i,j,k\},\{l\}\}$ when i, j and k are linked, and l is alone;

$B_{(ij,kl)} = \{\{i,j\},\{k,l\}$ when i, and j are linked, k and l are linked; $B_{(ij,k,l)} = \{\{i,j\},\{k\},\{l\}\}$ when i and j are linked, k is alone and l is alone; $B_{(ijkl)} = \{N\}$ when i, j, k and l are linked.

Similar notations for partitions will be used for the five players' case.

4.1 Games with four players

In this section we examine the games played by parties in the Czech Republic and in the Republic of Cameroon Parliaments. We shall compare the situation of the Czech Republic after the 2002 legislative elections with the situation in the Cameroonian Parliament between 1993 and 1997. The distribution of seats in each country is described below:

Party	Czech Republic		Cameroon	
	Name	Seats	Name	Seats
1	Democratic Social Party	70	Cameroon People's Democratic Movement	89
2	Civic Democratic Party	58	National Union for Democracy and Progress	67
3	Communist Party	41	Union of the Peoples of Cameroon	18
4	Christian Democratic Party	31	Movement for the Defence of the Republic	6
Ideological axis:	3-1-4-2		–	

Let us note in the Czech Parliament, that the minimal number of votes necessary for any coalition to win is 101, while in Cameroon's, this number is 91. Thus these Parliaments can respectively be described by quota games in the following way: $G_1 = [101{:}70, 58, 41, 31]$, and $G_2 = [91{:}\ 89, 67, 18, 6]$.

Notice that G_1 and G_2 both have the same winning coalitions: 12, 13, 14, 123, 124, 134, 234, 1234.

Now, a crucial difference in the two countries is the fact that because of the ideological configuration of the Czech Parliament, the formation of coalitions will depend on an assumption of single-peakedness over the set of parties. From left to the right, we have 3-1-4-2. More precisely, in the Czech Parliament, the Civic Democratic Party (2) and the Communist Party (3) alone cannot form a coalition, that is without 1 and 4. Also, the Christian Democratic Party (4) and the Communist Party (3) cannot form a coalition without 1. And finally, 1 and 2 cannot form a coalition without 4. For the Cameroonian case, no such assumption will be made since parties cannot be ranked along such a one-dimensional axis.

Given the elements above, we can now provide the measures of power, according to two of classical power indices. Values of Owen and Banzhaf, for each partition, are summarized in Appendix Table 3. Bold numbers in Appendix Table 3 correspond to the partition in which party 1 has maximal power. The set of all partitions obtained with the parliamentary method is given in Appendix Table 4. Now, using Owen and Banzhaf power indices, our purpose is to compare the partitions that will be selected depending on the way the selection is made; here we distinguish the two methods mentioned above: the parliamentary method and the core.

The following table summarizes the results:

	Czech Republic 2002–2006		Republic of Cameroon 1993–1997	
	Parliamentary	Core	Parliamentary	Core
Owen	$B_{(124,3)}$ $B_{(134,2)}$	$B_{(134,2)}$, $B_{(124,3)}$ $B_{(13,2,4)}$, $B_{(14,2,3)}$	$B_{(123,4)}$ $B_{(124,3)}$ $B_{(134,2)}$	$B_{(12,3,4)}$ $B_{(13,2,4)}$ $B_{(14,2,3)}$
Banzhaf	$B_{(14,2,3)}$	$B_{(13,2,4)}$ $B_{(14,2,3)}$	–	–

We can see that, when the power is measured by Owen's value:

i) The parliamentary method leads to the choice of partitions which are in the core:$B_{(124,\ 3)}$ and$B_{(134,2)}$ for the Czech parliament. The parliamentary method leads to the choice of partitions$B_{(123,4)}$, $B_{(124,3)}$, $B_{(134,2)}$ which are different from the choice of the core ($B_{(12,3,4)}$, $B_{(13,2,4)}$, $B_{(14,2,3)}$) for the Cameroonian parliament.

ii) $B_{(13,2,4)}$ is in the core for both the Czech and the Cameroonian parliaments, satisfies criterion 1 for both the Cameroonian parliament and Czech parliament but fails to satisfy the external stability.

$B_{(14,2,3)}$ is in the core for both the Czech and the Cameroonian parliaments, satisfies criteria 1 and 2 for the Czech parliament and fails to satisfy the internal stability condition. For the Cameroonian parliament$B_{(14,2,3)}$ satisfies criterion 1 and fails to satisfy internal and external stability.$B_{(12,3,4)}$ is only in the core for the Cameroonian parliament, satisfies criterion 1, but fails to satisfy the external and internal stability conditions. It is also a non valid partition for the Czech parliament due to ideological constraints.When the power is measured by the Banzhaf value:

i) For the Czech parliament, the core leads to the choice of partitions: $B_{(14,2,3)}$ and $B_{(13,2,4)}$.

The external stability of $B_{(14,2,3)}$ is due to the fact that, because of ideological constraints, $B_{(14,23)}$ is not a valid partition.

ii) For the Cameroonian parliament, the parliamentary method does not propose any partition because partitions $B_{(12,3,4)}$,$B_{(13,2,4)}$,$B_{(14,2,3)}$ fails to satisfy the external stability. The core of Cameroonian parliament is empty because of the presence of the partition $B_{(234,1)}$ which dominated the partitions suggested by core of the Czech Parliament. The partition $B_{(234,1)}$ is also dominated by $B_{(12,34)}$.

Our next question is as follows: how predictive are these methods to understand actual political behaviour?

i) Government {1, 4} is the one which was formed after the Czech legislative elections of 2002, with an Opposition made of parties 2 and 3 existing separately, that is partition $B_{(14,2,3)}$. This situation corresponds to the government composed of Democratic Social Party, Christian Democratic Party while Civic Democratic Party and the Communist Party are in Opposition.

This government is proposed by the core according to both indices of power. Let us note that each method proposes governments other than {1, 4}.

ii) After the legislative elections of 1993 in Cameroon, the first government which was formed was composed of the pair Cameroon People's Democratic Movement-Movement for the Defence of the Republic, what coincides with the proposal of the core according to Owen's value. Let us note that this government does not satisfy external stability according to Owen's value and Banzhaf value: the power of party 1 is maximum only if the Opposition is divided. However, two years later, the governmental coalition widened by the integration of Union of the Peoples of Cameroon. This new government is more in conformity with the theoretical forecasts of the parliamentary method according to Owen's value.

4.2 The German parliament game

The distribution of seats in the German parliament (2005–2009) country is described as follows:

Party	Name	Seats
1	Democratic Christian Union –Democratic Social Union	226
2	Democratic Social Party	222
3	Liberal Party	61
4	Party of Left	54
5	Greens	51
Ideological axis: 4-5-2-1-3		

This Parliament is described as the game $G_3 = [308: 226, 222, 61, 54, 51]$

Winning coalitions are the following ones: 12, 123, 124, 125, 134, 135, 145, 234, 235, 245, 1345 and 2345.

Now, German parties can be ranked on an ideological axis from left to the right: 4-5-2-1-3. This implies that the only admissible partitions are the ones in Table 1, which simultaneously indicates the power of every admissible partition, according to the two power indices.

The parliamentary method and the core yield the results summarized below:

	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Core
Owen	$B_{(12,3,45)}$, $B_{(123,45)}$, $B_{(12,3,4,5)}$	$B_{(12,3,45)}$, $B_{(12,3,4,5)}$	$B_{(12,3,45)}$, $B_{(123,45)}$, $B_{(12,3,4,5)}$	$B_{(12,3,45)}$, $B_{(12,3,4,5)}$	$B_{(12,3,45)}$, $B_{(123,45)}$, $B_{(12,3,4,5)}$
Banzhaf	$B_{(12,3,45)}$, $B_{(12,3,4,5)}$	$B_{(12,3,45)}$, $B_{(12,3,4,5)}$	$B_{(12,3,45)}$, $B_{(12,3,4,5)}$	$B_{(12,3,45)}$, $B_{(12,3,4,5)}$	$B_{(12,3,45)}$, $B_{(12,3,4,5)}$, $B_{(1,3,245)}$

The parliamentary method along with Owen's value leads to coalition {1, 2}, parties 3, 4 and 5 being divided in the opposition. And this is exactly the government that emerged from the 2005 legislative elections.

Table 1 Power indices in the German parliament game

Partitions	Owen value	Banzhaf index
$B_{(12345)}$	0.30; 0.30; 0.13; 0.13; 0.13;	0.28; 0.28; 0.14; 0.14; 0.14;
$B_{(1235,4)}$	0.33; 0.33; 0.16; 0; 0.16;	0.33; 0.33; 0.16; 0; 0.16;
$B_{(1245,3)}$	0.33; 0.33; 0; 0.16; 0.16;	0.33; 0.33; 0; 0.16; 0.16;
$B_{(123,45)}$	**0.50**; 0.50; 0; 0; 0;	**0.50**; 0.50; 0; 0; 0;
$B_{(123,4,5)}$	0.44; 0.44; 0.12; 0; 0;	0.40; 0.40; 0.20; 0; 0;
$B_{(125,3,4)}$	0.44; 0.44; 0; 0; 0.12;	0.40; 0.40; 0; 0; 0.20;
$B_{(12,3,4,5)}$	**0.50**; 0.50; 0; 0; 0;	**0.50**; 0.50; 0; 0; 0;
$B_{(12,45,3)}$	**0.50**; 0.50; 0; 0; 0;	**0.50**; 0.50; 0; 0; 0;
$B_{(13,245)}$	0; 0.33; 0; 0.33; 0.33;	0; 0.33; 0; 0.33; 0.33;
$B_{(1,3,245)}$	0; 0.44; 0; 0.28; 0.28;	0; 0.50; 0; 0.25; 0.25;
$B_{(1,2,3,4,5)}$	0.30; 0.30; 0.13; 0.13; 0.13;	0.28; 0.28; 0.14; 0.14; 0.14;

Also, the proposal of the core according to Owen's value coincides with the government which was really formed: $B_{(12,3,4,5)}$ and $B_{(12,3,45)}$.

This situation corresponds to the government composed of Democratic Social Party, Democratic Christian Union and Democratic Social Union; while Liberal Party, Party of Left and the Greens are on the opposition. Let us note that the core proposes governments other than {1, 2} more precisely the government $B_{(123,45)}$.

The parliamentary method along with the index of Banzhaf leads to coalition {1, 2}, parties 3, 4 and 5 being divided in the opposition. And again this is the government that emerged from the 2005 legislative elections.

The core according to the index of Banzhaf makes the same proposal but it adds the possibility that parties 2, 4, 5 have to form a government and parties 1 and 3 are divided in the opposition: $B_{(1,3,245)}$.

Partitions $B_{(1,3,245)}$ are not selected by the parliamentary method because it does not give maximum power to party 1. This situation corresponds to the government composed of Democratic Social Party, Party of Left, and the Greens; while Democratic Christian Union and Democratic Social Union and Liberal Party divided in the Opposition.

4.3 The Catalonian parliament game

The distribution of seats in the Catalonian parliament (1980–1984) is described as follows:

Party	Name	Seats
1	Convergence-Union	43
2	Catalan Socialist Party	33
3	Catalan Communist Party	25
4	Catalan Centrist	18
5	Catalan Republic Left	14
Ideological axis:3-2-5-4-1		

The corresponding game is $G_4 = [68: 43, 33, 25, 18, 14]$. The winning coalitions of G_4 are the following ones: 12, 13, 145, 123, 124, 134, 125, 135, 234, 235, 1234, 1245, 1235, 1345, 2345 and 12345.

The measure of power according Owen and Banzhaf values are given in Table 2 below:

The core and parliamentary methods then yield:

	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Core
Owen	$B_{(1245,3)}$	$B_{(1245,3)}$	$B_{(1245,3)}$	$B_{(1245,3)}$	$B_{(14,235)}$
Banzhaf	$B_{(145,2,3)}$	–	–	–	$B_{(14,235)}$

Table 2 Power indices in the Catalonian parliament game

Partitions	Owen value	Banzhaf index
$B_{(12345)}$	0.40; 0.23; 0.23; 0.066; 0.066	0.38; 0.23; 0.23; 0.08; 0.08
$B_{(1245,3)}$	**0.58**; 0.25; 0; 0.08; 0.08;	0.50; 0.30; 0; 0.10; 0.10
$B_{(2345,1)}$	0; 0.42; 0.42; 0.08; 0.08;	0; 0.375; 0.375; 0.125; 0.125
$B_{(145,23)}$	0.33; 0; 0; 0.33; 0.33	0.33; 0; 0; 0.33; 0.33
$B_{(145,2,3)}$	0.56; 0; 0; 0.22; 0.22	**0.72;**0; 0; 0.14; 0.14
$B_{(1,3,245)}$	0.33; 0.23; 0.33; 0.055; 0.055	0.31; 0.23; 0.31; 0.07; 0.07
$B_{(14,235)}$	0; 0.33; 0.33; 0; 0.33	0; 0.33; 0.33; 0; 0.33
$B_{(1,235,4)}$	0; 0.39; 0.39; 0; 0.22	0; 0.43; 0.43; 0; 0.14
$B_{(14,23,5)}$	0.16; 0.16; 0.16; 0; 16.0.33	0.16; 0.16; 0.16; 0; 16.0.33
$B_{(1,23,4,5)}$	0.16; 0.25; 0.25; 0.16; 0.16	0.16; 0.25; 0.25; 0.16; 0.16
$B_{(14,2,3,5)}$	0.42; 0.16; 0.16; 0.08; 0.16	0.42; 0.16; 0.16; 0.08; 0.16
$B_{(1,2,3,4,5)}$	0.40; 0.23; 0.23; 0.066; 0.066	0.38; 0.23; 0.23; 0.08; 0.08

In the Catalan case, the actual government was composed of parties 1, 4 and 5, which are the Convergence-Union, the Catalan Centrist and the Catalan Republican Left.

The core according Owen's value and Banzhaf value's also selects partitions $B_{(14,235)}$. This situation corresponds to the government composed of the Catalan Socialist party, the Catalan Communist Party and the Catalan Republican Left. The opposition is composed of the Convergence-Union and the Catalan Centrist, united or divided.

The parliamentary method according to Owen value's selects partition$B_{(1245,3)}$. This situation corresponds to the government composed of the Convergence-Union, the Catalan Centrist, and the Catalan Republican Left. This government is larger than the government which was really formed.

Let us now compare with Owen and Carreras (1988) results. The Catalan Communist Party is alone in opposition.

Owen and Carreras supposed that certain coalitions can be formed, they evaluated they power of each party in these partitions.

They supposed that the Catalan Socialist Party and the Catalan Communist Party form a coalition each of the other parties being alone: it is the partition$B_{(1,\ 23,\ 4,\ 5)}$ in our study.

The value of Owen of each party in this partition is$B_{(1,23,4,5)}$ = [0.16; 0.25; 0.25; 0.16; 0.16].

The index of Banzhaf of each party in this partition is $B_{(1,23,4,5)}$ = [0.16; 0.25; 0.25; 0.16; 0.16].

Owen and Carreras supposed that the convergence-Union party and the Catalan Republican Left form a coalition, each of the other parties being alone: this gives partition $B_{(14,\ 2,\ 3,\ 5)}$ in our study.

The value of Owen of each party in this partition is: $B_{(14,2,3,5)}$ = [0.42; 0.16; 0.16; 0.08; 0.16].

The index of Banzhaf of each party in this partition is: $B_{(14,2,3,5)}$ = [0.42; 0.16; 0.16; 0.08; 0.16].

If the Catalan Socialist Party and the Communist party form a coalition on the one hand, the Convergence-Union and the Catalan Centrist forms also a coalition on the other hand, the Catalan Republican Left being alone: it is the partition $B_{(14,\ 23,\ 5)}$ in our study.

The value of Owen of each party in this partition is: $B_{(14,23,5)}$ = [0.16; 0.16; 0.16; 0.16; 0.33].

The index of Banzhaf of each party in this partition is: $B_{(14,23,5)}$ = [0.16; 0.16; 0.16; 0.16; 0.33].

Finally Owen and Carreras suppose that the Catalan Socialist Party and the Catalan Communist Party form a single coalition, they associate the Catalan Republic Left.

Our study does not consider this political configuration, because it supposes that parties 2 and 3 must vote together each time, we cannot have 253, or 352 but we can have 235 or 325.

Our study supposes on the other hand that parties 2, 3 and 5 form only one coalition and the two other parties being divided: it is the partition $B_{(1,235,4)}$.

Owen's value of each party in partition $B_{(1,235,4)}$ is: [0; 0.39; 0.39; 0; 0.22].

The index of Banzhaf of each party in partition $B_{(1,235,4)}$ is: [0; 0.43; 0.43; 0.0.14].

When Opposition is united Owen's value for each party in partition $B_{(14,235)}$ is: [0; 0.33; 0.33; 0; 0.33], and the index of Banzhaf of each party in partition $B_{(14,235)}$ = [0; 0.33; 0.33; 0; 0.33].

In the same manner the Convergence-Union and the Catalan Centre form a single coalition by associating the Catalan Republican Left, our study does not consider this political configuration, because it supposes that 1 and 4 votes together each time, we cannot have 154, 451 but we can have 145 or 415.

Our study supposes on the other hand that parties 1, 4 and 5 form only one coalition and the two other parties being divided: which gives partition $B_{(145,2,3)}$.

Owen's value of each party in partition is: $B_{(145,2,3)}$ = [0.56; 0; 0; 0.22; 0.22] and the index of Banzhaf of each party in partition is: $B_{(145,2,3)}$ = [0.72; 0; 0; 0.14; 0.14].

When Opposition is united Owen's value of each party in partition $B_{(145,23)}$ is: [0.33; 0; 0; 0.33; 0.33], and the index of Banzhaf of each party in partition $B_{(145,23)}$ is: [0.33; 0; 0; 0.33; 0.33].

5 Conclusion

This paper was devoted to the study of the possibility of a stable government after general elections. We can notice that the solution of the game will always be a partition of N. The corresponding political configuration is a situation where there are one the one hand a government and on the other hand an opposition, which can be united or divided.

We successively consider 4-player and 5-player games. The two 4-player games are equivalent in the sense that although the weights and the quotas are different, they have exactly the same winning coalitions; but they differ in that in one of them—the Czech Republic—political parties can be ranked along an ideological axis (which corresponds to single-peakedness), whereas in the other one—the Republic of Cameroon—there is no such possibility. It appears that the presence of an ideological axis increases the tendency to stability: the proportion of stable coalitions (the number of all stable coalitions out of the total number of all possible coalitions) is higher for the Czech Republic than for the Republic of Cameroon; and this is true whatever the underlying power index. In our two 5-player games, all parties can be ranked along an ideological axis; however, the two games do not have the same set of winning coalitions. This shows that, under single-peakedness, stability highly depends on other parameters of the game. Let us first notice that the parliamentary method gives an advantage to the party that has the largest number of seats, that is the party with the largest weight (and which is then in charge of forming the government).

Ultimately, note that the solutions proposed by party 1 can be disapproved by the other parties, especially in a dynamic approach where each party makes the same reasoning as party 1 and looks for the partitions in which it belongs to a stable government. The core gives this possibility to all political parties. With the core, party 1 can be found in the opposition although it has the largest number of seats in the parliament, in particular when the government proposed by the other parties dominates the government proposed by party 1.

The choice of the optimal partition depends on both the underlying power index and the description of the ideological landscape of the political parties in each country. In other words, the choice of the optimal partition also depends on the hypothesis of single-peakedness over the set of parties.

Finally, it appears that when two different power indices lead to the same stable partitions with party 1, these partitions are equivalent although the power that each index assigns to party 1 is different.

The results in the paper show that the analysis above is to a large extent in accordance with what actually happened during the periods under consideration.

The scope of this paper is restricted to games with four or five players due to illustrations, although the framework is general; with more players the number of partitions grows considerably; nevertheless, it should be interesting, at least under the hypothesis of single-peakedness, to consider games with a greater number of players.

Acknowledgments We are grateful to Guillermo Owen for stimulating discussions and to an anonymous referee for very helpful comments and suggestions.

Appendix

Table 3 Power indices for the two 4-player parliaments

Partitions	Owen's value		Banzhaf index	
	With single-peakedness Czech Republic	Without single-peakedness Republic of Cameroon	With single-peakedness Czech Republic	Without single-peakedness Republic of Cameroon
$B_{(1234)}$	0.50; 0.16; 0.16; 0.16	0.50; 0.16; 0.16; 0.16	0.50; 0.16; 0.16; 0.16	0.50; 0.16; 0.16; 0.16
$B_{(123,4)}$	–	**0.66;** 0.16; 0.16; 0	–	0.60; 0.20; 0.20; 0
$B_{(124,3)}$	**0.66**; 0.16; 0; 0.16	**0.66**; 0.16; 0; 0.16	0.60; 0.20; 0.20; 0	0.60; 0.20; 0.20; 0
$B_{(134,2)}$	**0.66**; 0; 0.16; 0.16	**0.66**; 0.16; 0; 0.16	0; 60; 0; 0.20; 0.20	0; 60; 0; 0.20; 0.20
$B_{(234,1)}$	–	0; 0.33; 0.33; 0.33	–	0; 0.33; 0.33; 0.33
$B_{(12,34)}$	–	0.50; 0.50; 0; 0	–	0.50; 0.50; 0; 0
$B_{(12,3,4)}$	–	**0.66**; 0.33; 0; 0	–	**0.75**; 0.25; 0; 0
$B_{(1,2,34)}$	–	0.33; 0.33; 0.16; 0.16	–	0.33; 0.33; 0.16; 0.16
$B_{(13,24)}$	0.50; 0; 0.50; 0	0.50; 0; 0.50; 0	0.50; 0; 0.50; 0	0.50; 0; 0.50; 0
$B_{(13,2,4)}$	**0.66**; 0; 0.33; 0	**0.66**; 0; 0.33; 0	0.75; 0; 0.25; 0	**0.75**; 0; 0.25; 0
$B_{(1,3,24)}$	0.33; 0.16; 0.33; 0.16	0.33; 0.16; 0.33; 0.16	0.33; 0.16; 0.33; 0.16	0.33; 0.16; 0.33; 0.16
$B_{(14,23)}$	–	0.50; 0; 0; 0.50	–	0.50; 0; 0; 0.50
$B_{(14,2,3)}$	**0.66**; 0; 0; 0.33	**0.66**; 0; 0; 0.33	0.75; 0; 0; 0.25	**0.75**; 0; 0; 0.25
$B_{(1,4,23)}$	–	0.33; 0.16; 0.16; 0.33	–	0.33; 0.16; 0.16; 0.33
$B_{(1,2,3,4)}$	0.50; 0.16; 0.16; 0.16	0.50; 0.16; 0.16; 0.16	0.50; 0.16; 0.16; 0.16	0.50; 0.16; 0.16; 0.16

Table 4 Partitions for the two 4-player parliaments under the parliamentary method

Power index		Criterion 1	Criterion 2	Criterion 3	Criterion 4
Owen	SP	$B_{(13,2,4)}$, $B_{(14,2,3)}$	$B_{(124,3)}$	$B_{(124,3)}$	$B_{(124,3)}$
	no SP	$B_{(123,4)}$, $B_{(124,3)}$	$B_{(123,4)}$	$B_{(123,4)}$	$B_{(123,4)}$
		$B_{(134,2)}$, $B_{(12,3,4)}$	$B_{(124,3)}$	$B_{(124,3)}$	$B_{(124,3)}$
		$B_{(13,2,4)}$, $B_{(14,2,3)}$	$B_{(134,2)}$	$B_{(134,2)}$	$B_{(134,2)}$
Banzhaf	SP	$B_{(13,2,4)}$, $B_{(14,3,4)}$	$B_{(14,2,3)}$	$B_{(14,2,3)}$	$B_{(14,2,3)}$
	no SP	$B_{(12,3,4)}$		$B_{(12,3,4)}$	
		$B_{(13,2,4)}$	–	$B_{(13,2,4)}$	–
		$B_{(14,2,3)}$		$B_{(14,2,3)}$	

References

Axelrod, R. (1970). *Conflict of interest* Chicago:Markham.

Holler, M. J. (2001). G. Owen (Eds.) *Power indices and coalition formation.* Boston/Dordrecht/London: Kluwer Academic Publishers.

Leiserson, M. (1966). Coalitions in politics Ph.D. Dissertation. Yale University, New Haven.

Owen, G., & Carreras, F. (1988). Evaluation of the Catalonian Parliament 1980–1984. *Mathematical Social Sciences, 15*(1), 87–92.

Owen, O. (1977). Values of games with a priori unions. In R. Henn, O. Moeschlin. (Eds.), *Mathematical Economics and Game Theory*(pp. 76–88). Berlin:Springer-Verlag.

Owen, G. (1981). Modification of the Banzhaf-Coleman index for games with a priori unions. In M. J. Holler. (Ed.), *Voting and voting Power*. (pp. 232–238). Wurzburg: Physical-Verlag.
Shapley, L. S. (1953). A value for *n*-person games, Annals of Mathematics Studies, Study 28. (pp. 307–317). Princeton University Press.
Taylor, M. (1971). Mathematical political theory. *British Journal of Political Science, 1*(3), 339–382.