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Chirped super–Gaussian and super–sech pulse perturbation of nonlinear Schrödinger's equation with quadratic–cubic nonlinearity by variational principle

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ABSTRACT

We apply variational method to the perturbed nonlinear Schrödinger equation having quadratic-cubic form of nonlinearity, to study localized optical pulses. Super-Gaussian and super-sech solitons are used as envelopes for the trial function. Numerical simulations are presented for specific values of the Gaussian and super-sech pulse parameters. The impact of the quadratic-cubic terms on the evolution for different parameters is assessed. In general, when the nonlinear quadratic and cubic coefficients increase, the frequency of the oscillations of the collective variables also increases.

1. Introduction

Localized optical pulses, or optical solitons, are the fundamental bit carriers in fiber optic telecommunication systems. There exists a wide variety of aspects of their propagation that have been addressed and an abundance of results have been reported [1–31]. These include topics from soliton perturbation theory, integrability aspects, variational principle, collective variables, quasi-stationarity, and many others. In these contexts, a wide range of pulse dynamics have been studied. This paper revisits the variational principle for the governing nonlinear Schrödinger equation (NLSE). The law of refractive index that will be taken up in this work is of the quadratic–cubic (QC) type, which is of particular relevance to fiber communications. This kind of nonlinearity first appeared in 1994 and later resurfaced again in 2011, after a long hiatus [7,8]. In [8] the authors investigated exact soliton solutions of nonlinear Schrödinger equation with quadratic-cubic nonlinearity. They used Jacobi elliptic functions to study optical beams. As a result, dark and bright solitons have been obtained and their stability analyzed. Subsequently, this law of nonlinearity in optical fibers was explored in a number of papers that were published in a variety of journals.

This manuscript is going to handle NLSE with QC nonlinearity as well, having deterministic perturbation terms. In the current work, in the presence of such deterministic perturbations, it is proposed to use a variational method with a numerical scheme as the most effective in treating a complex dynamical system with six soliton wave parameters. The parameters are the soliton amplitude, width, chirp, center position, speed and phase variation. The paper adopts the mathematical methodology in the form of a Lagrangian variational (LV) method. The variational technique is well known and used in several areas and it is particularly useful in nonlinear optics for describing the

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propagation of short pulses in nonlinear media. The variational formulation was first utilized in 1977, to describe pulse propagation in nonlinear diffracting media. Its use to describe nonlinear dispersive media such as optical fibers was first realized by Anderson in 1983. A variational approach can be quite accurate even when the effect of nonlinearity is large. The accuracy depends crucially on the ansatz solution chosen to describe the evolution of the optical field experiencing nonlinearity [9]. With a touch of necessary numerical analysis, the dynamical system developed in this way offers a good alternative to following the evolution of solitons through the full numerical solution of partial differential equations in question.

The pulse types that will be incorporated in this work are of the super–Gaussian and super–sech form. The Gaussian and the sech pulses are the fundamental soliton solutions governed by the generalized NLSEs. These models have produced a better description of coherent radiation, like the pulsed laser beams. Added nonlinear terms to the standard Schrödinger equation require taking into account several parameters in the trial function. To make further progress in studies of solitons in optical fibers or other forms of waveguides that maintain the QC form of nonlinearity, the LV technique serves as an essential commodity. The aim of this work is to study the impact of nonlinear effects in order to be able to optimize the transmission of information in the fiber. Different parameters are chosen to obtain solutions presenting the characteristics of an optical soliton and could be used in telecommunications, in industry, in the design of new waveguides, as well as in the choice of incident pulses that would optimize the transmission of information.

This contribution is organized as follows. The governing model is first described by using the appropriate NLSE. Then the LV method and the RK4 numerical method are presented as the tools to solve the perturbed NLSE. The last section is devoted to numerical simulations. The results obtained in this paper could make a significant impact on the future work in nonlinear optics of fibers and many applications in the telecommunications industry. This work could be utilized for optimization of the transmission of data in optical fibers.

1.1. Governing model

The dimensionless perturbed NLSE with the QC form of nonlinearity is formulated as [12,16]:

$$iq_z + aq_{tt} + \left(b_1|q| + b_2|q|^2\right)q = i\alpha q_t + i\lambda\left(|q|^2 q\right)_t + i\theta\left(|q|^2\right)_t q. \tag{1}$$

In addition to the equation one should specify the boundary and initial conditions, $q(t, z = 0)$. Here, $i = \sqrt{-1}$, and the dependent variable is $q(z,t)$ that defines the slowly-varying wave profile. Here, z $(z \geq 0)$ and t (real) are the independent variables that stand for the spatial and temporal coordinates, respectively. The first term stands for the temporal evolution and a is the coefficient of group velocity dispersion (GVD), while b_1 and b_2 together comprise the QC form of nonlinearity. On the right hand side, α represents the inter-modal dispersion (IMD), while λ is the self-steepening (SS) coefficient and θ stands for the nonlinear dispersion (ND).

2. Methodology

The idea of the LV method is to associate a collective variable with each degree of freedom in an equivalent mechanical system. Each collective variable is introduced to describe a fundamental physical quantity of the soliton (its amplitude, temporal position, width at half maximum, chirp, etc.). To this end, one determines the Lagrangian of the system, which is a function of the exact impulse field $\psi(t,z)$ whose analytical expression is known: $\psi(t,z) = f(t, x_1, x_2, ...x_n)$. Then using Euler-Lagrange equations one finds a system of coupled variational equations as functions of the collective variables. These equations are solved by a numerical method (in our case, $RK4$). The choice of the ansatz ψ is a crucial step in the implementation of a variational method, because it should represent the nature of the incident and propagating impulse field [17,24] as close as possible, yet be of the form that allows a tractable analytical and numerical treatment. The interest in this method increases with the ease in which it makes possible to obtain the fixed points, as well as the cartography of various types of solutions, which usually reduces by several orders of magnitude the computation volume and time required to obtain the same type of results numerically. This the rationale behind our decision to use this method, coupled with the simple Runge–Kutta numerical method.

The Lagrangian density constitutes the initiation of the Lagrangian Variational Method. The Lagrangian L of this system without the perturbation terms may be cast into the form:

$$L = \int_{-\infty}^{+\infty} \frac{i}{4}\left(q_z q^* - q_z^* q\right) - \frac{a}{2}\mid q_t \mid^2 + \frac{b_1}{3}\mid q \mid^3 + \frac{b_2}{4}\mid q \mid^4 dt. \tag{2}$$

Now, we assume that the solution of this system, a chirped pulse, is taken to be

$$q(X_J, t) = X_1 f\left(\frac{t - X_2}{X_3}\right)\exp\left[i\left(\frac{X_4}{2}(t - X_2)^2 + X_5(t - X_2) + X_6\right)\right], \qquad J = \{1, 2...6\} \tag{3}$$

where f stands for the shape of the pulse, in this paper chosen to be the super-sech ($f(\tau) = \operatorname{sech}^m(\tau)$) or the super–Gaussian ($\exp(-\tau^{2m})$). Here, the parameters X_j with $j = 1, 2...6$ respectively, stand for the amplitude of the soliton, the center of the pulse, the width of the pulse, the chirp, the frequency, and the phase of the pulse, all the functions of z. Also, m is the parameter of super-sech or super–Gaussian pulses that controls their steepness and spectral width.

Concerning the pulse parameters, the Lagrangian variational principle leads to a number of evolution equations. It is worth noting that this approach provides only approximate solutions and it is not sensitive to continuum radiation sourced loss of energy, pulse shape change and the damping of amplitude oscillations. With this in mind, defining the following integral is convenient for future calculations:

$$I_{i,j,k} = \int_{-\infty}^{+\infty} \tau^i f^j(\tau)\left(\frac{df}{d\tau}\right)^k d\tau, \tag{4}$$

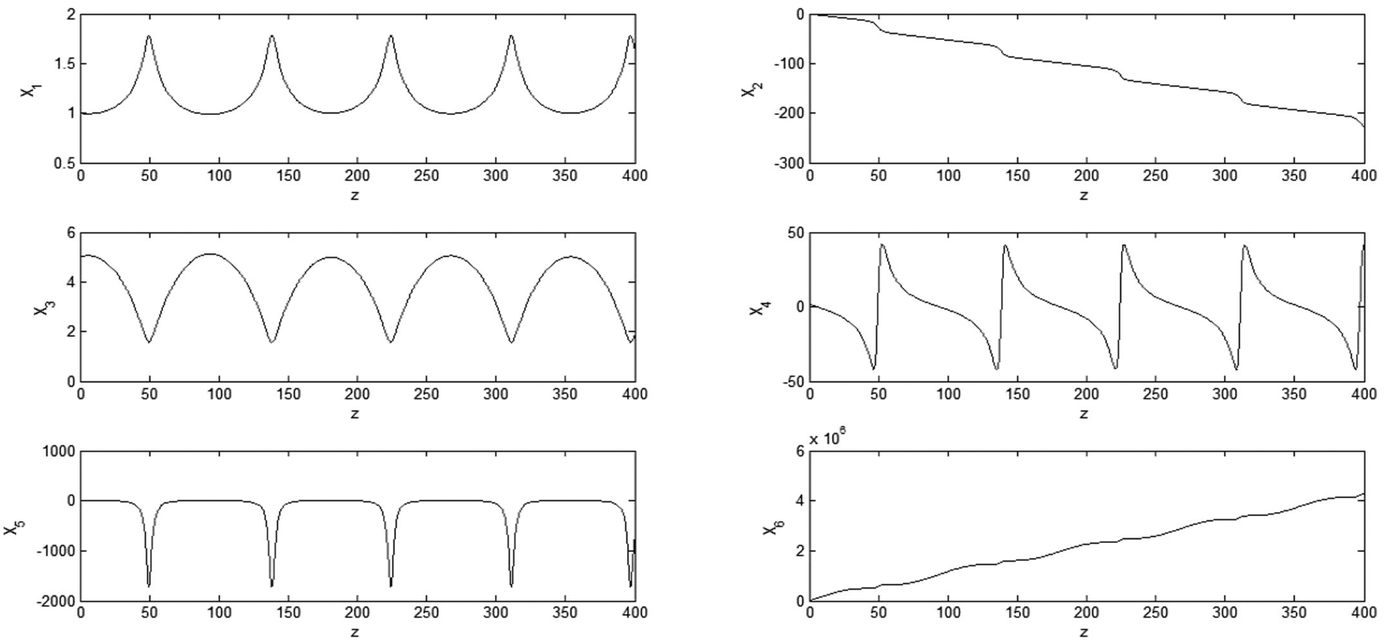

Fig. 1. Variation of super-Gaussian pulse parameters, $m = 1$ (X_1-soliton amplitude, X_2-center position of the soliton, X_3-pulse width, X_4-soliton chirp, X_5-soliton frequency, X_6-soliton phase).

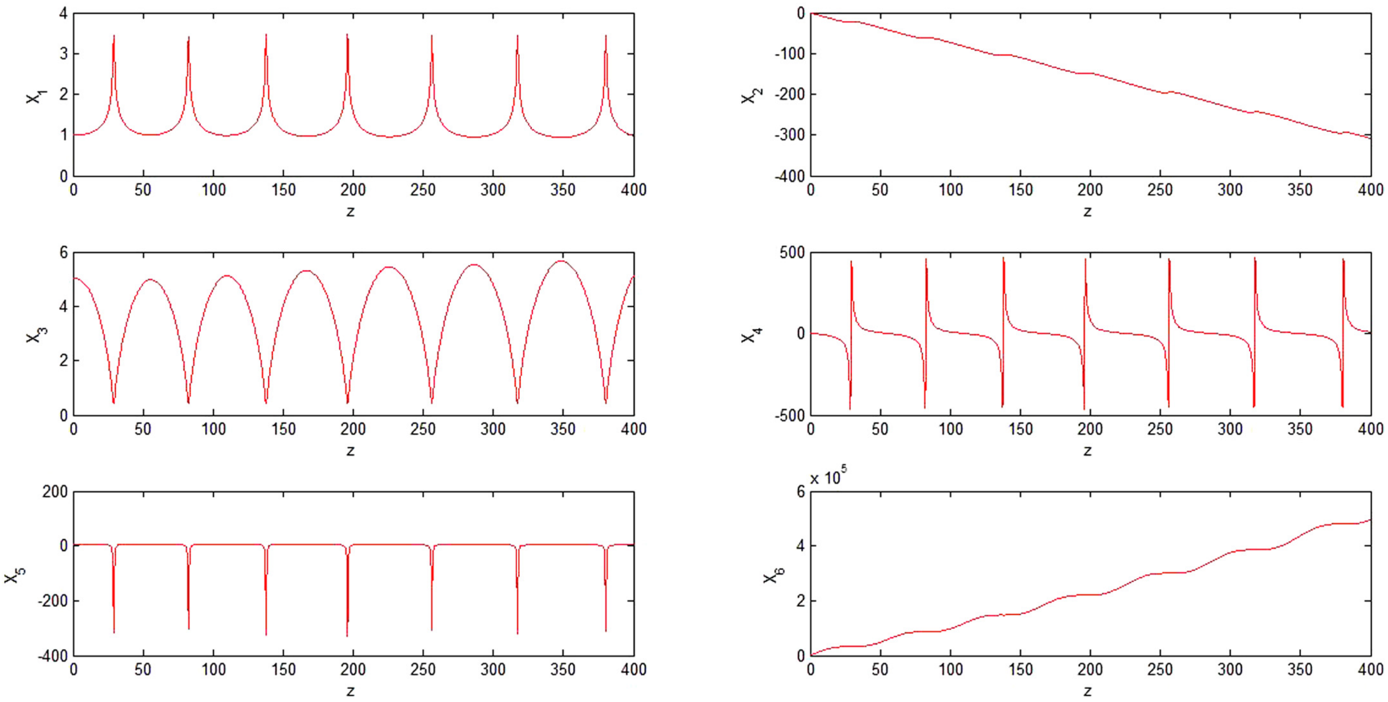

Fig. 2. Variation of super-sech pulse parameters, $m = 2$ (X_1-soliton amplitude, X_2-center position of the soliton, X_3-pulse width, X_4-soliton chirp, X_5-soliton frequency, X_6-soliton phase).

where i, j and k are non-negative integers. Next, employing (3), the Lagrangian (2) changes to:

$$L = -\frac{1}{4}X_1^2X_4'X_3^3I_{2,2,0} + \frac{1}{2}(X_1^2X_5X_2'X_3I_{0,2,0} - \frac{1}{2}X_1^2X_6')X_3I_{0,2,0} + \frac{b_1}{3}X_1^3X_3I_{0,3,0} + \frac{b_2}{4}X_1^4X_3I_{0,4,0} - \frac{1}{2}\frac{X_1^2a\left(I_{0,0,2} + X_3^2X_5^2I_{0,2,0} + X_3^4X_4^2I_{2,2,0}\right)}{X_3}. \tag{5}$$

We now consider the perturbed system whose equation in general is given by:

$$iq_z + aq_{tt} + (b_1|q| + b_2|q|^2)q = i\epsilon R\left[f, f^*\right] \tag{6}$$

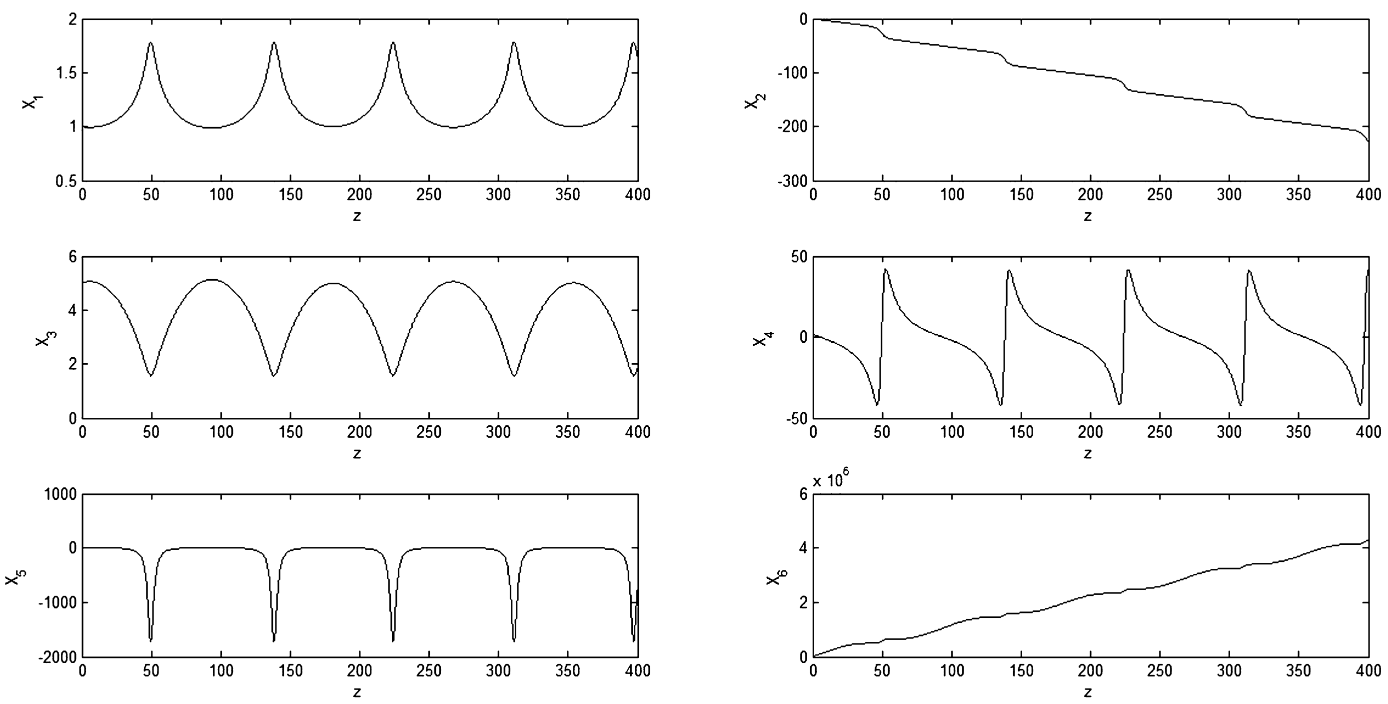

Fig. 3. Variation of super-Gaussian pulse parameters, $m = 2$ (X_1-soliton amplitude, X_2-center position of the soliton, X_3-pulse width, X_4-soliton chirp, X_5-soliton frequency, X_6-soliton phase).

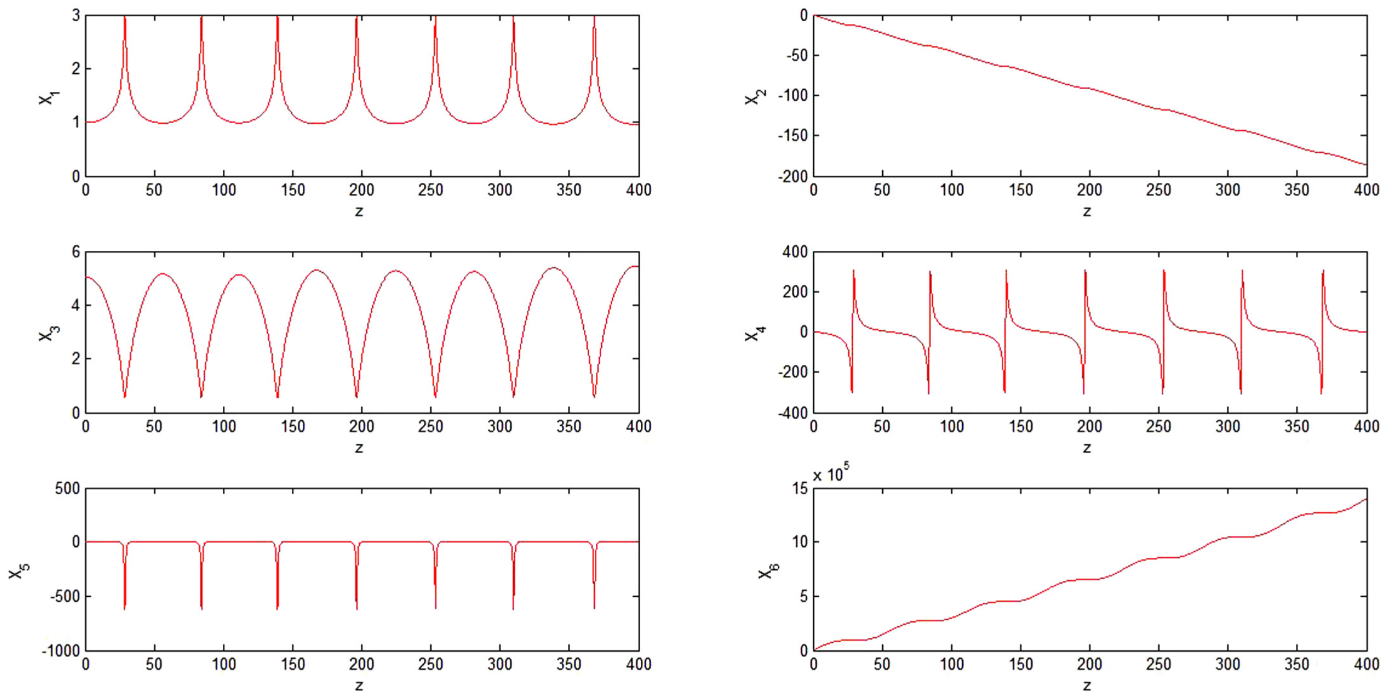

Fig. 4. Variation of super-sech pulse parameters, $m = 4$ (X_1-soliton amplitude, X_2-center position of the soliton, X_3-pulse width, X_4-soliton chirp, X_5-soliton frequency, X_6-soliton phase).

where ϵ is the perturbation parameter, which might be caused by the quasi-monochromaticity [17] and is called the relative width of the spectrum. Additionally, in (6), R includes different perturbation terms. Under the consideration of perturbation terms, the Euler Lagrange equations modify to:

$$\frac{\partial L}{\partial X_j(z)} - \frac{d}{dz}\frac{\partial L}{\partial \dot{X}_j(z)} = i\epsilon \int_{-\infty}^{+\infty} \left(R\frac{\partial q^*}{\partial X_j} - R^*\frac{\partial q}{\partial X_j} \right) dt \tag{7}$$

and the perturbation is chosen as:

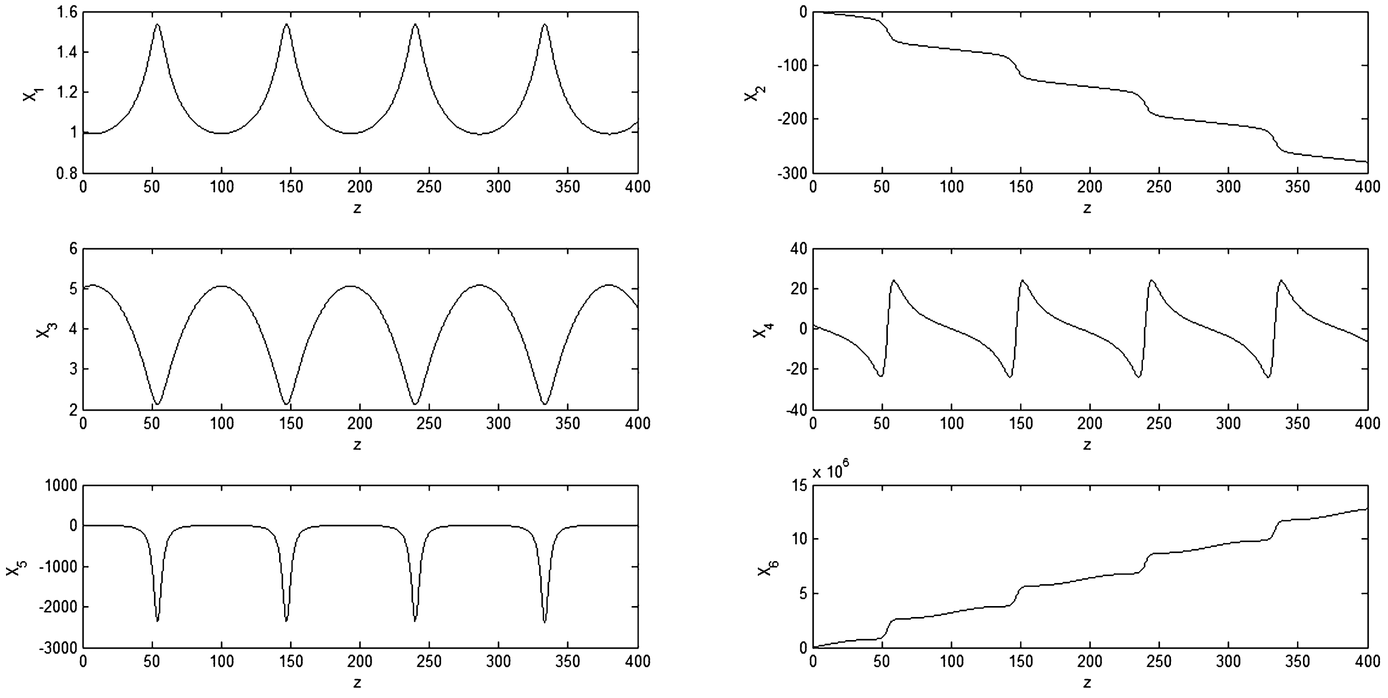

Fig. 5. Variation of super-Gaussian pulse parameters, $m = 4$ (X_1-soliton amplitude, X_2-center position of the soliton, X_3-pulse width, X_4-soliton chirp, X_5-soliton frequency, X_6-soliton phase).

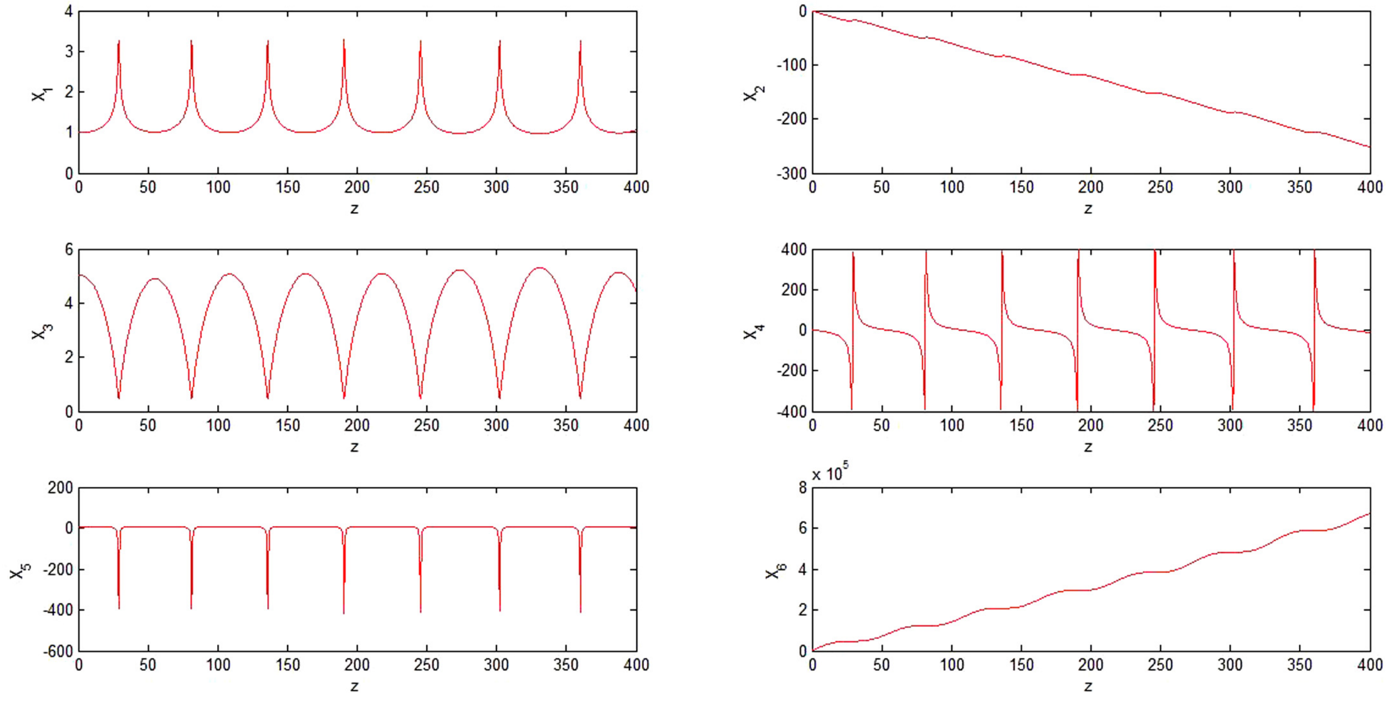

Fig. 6. Variation of super-sech pulse parameters, $m = 8$ (X_1-soliton amplitude, X_2-center position of the soliton, X_3-pulse width, X_4-soliton chirp, X_5-soliton frequency, X_6-soliton phase).

$$R = i\alpha q_t + i\lambda(|q|^2 q)_t + i\theta(|q|^2)_t q. \tag{8}$$

By setting,

$$\psi_j = i\epsilon \int_{-\infty}^{+\infty} \left(R\frac{\partial f^*}{\partial X_j} - R^* \frac{\partial f}{\partial X_j} \right) \tag{9}$$

and substituting the expression of the average Lagrangian given in equation (5) with the ansatz function f as ψ_j, the following adiabatic evolution equations are reached:

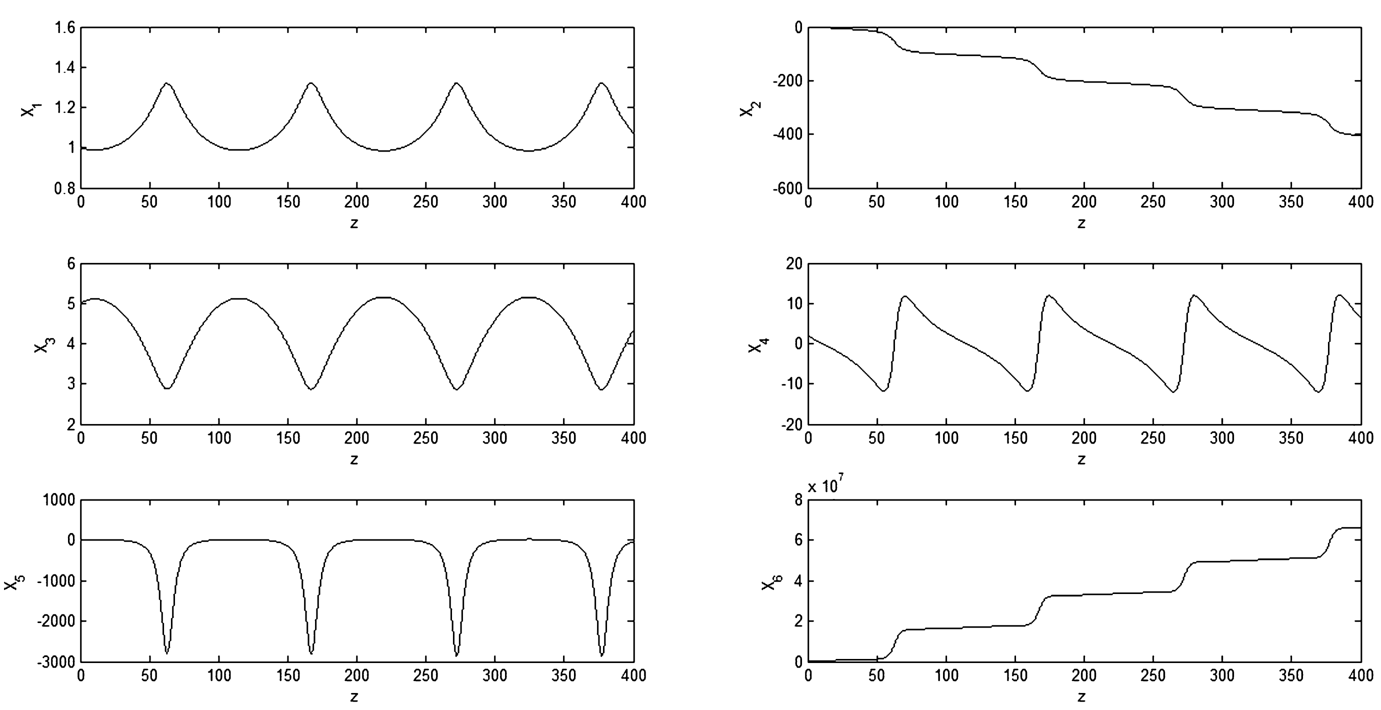

Fig. 7. Variation of super-Gaussian pulse parameters, $m = 6$ (X_1-soliton amplitude, X_2-center position of the soliton, X_3-pulse width, X_4-soliton chirp, X_5-soliton frequency, X_6-soliton phase).

$$\dot{X}_1 = -\frac{1}{2}\frac{2aI_{0,2,0}I_{2,2,0}X_1^2X_3^3 - 3I_{2,2,0}\psi_6X_3^2 + 2I_{0,2,0}\psi_4}{I_{0,2,0}I_{2,2,0}X_1X_3^3} \tag{10}$$

$$\dot{X}_2 = \frac{2\left(aI_{0,2,0}X_1^2X_3X_5 + \psi_5\right)}{I_{0,2,0}X_1^2X_3} \tag{11}$$

$$\dot{X}_3 = \frac{2aI_{0,2,0}I_{2,2,0}X_3^3X_4X_1^2 - \psi_6qX_3^2 + I_{0,2,0}\psi_4}{I_{0,2,0}I_{2,2,0}X_3^2X_1^2} \tag{12}$$

$$\dot{X}_4 = \frac{-12aI_{2,2,0}X_3^4X_4^2X_1^2 - 3I_{0,4,0}b_2X_1^4X_3^2 - 2I_{0,3,0}b_1X_1^3X_3^2 + 12aI_{0,0,2}X_1^2 + 6\psi_1X_1X_3 - 12\psi_3X_3^2}{6I_{2,2,0}X_1^2X_3^4} \tag{13}$$

$$\dot{X}_5 = -\frac{2aI_{0,2,0}I_{2,2,0}X_1^2X_3^3X_4X_5 + 4I_{2,2,0}\psi_2X_1X_3^2 + I_{2,2,0}\psi_6X_5X_3^2 + 2I_{0,2,0}\psi_4X_5}{I_{0,2,0}I_{2,2,0}X_1^2X_3^3} \tag{14}$$

$$\dot{X}_6 = -\frac{-12aI_{0,2,0}X_1^2X_3^2X_5^2 - 15I_{0,4,0}b_2X_1^4X_3^2 - 14I_{0,3,0}b_1X_1^3X_3^2 + 24aI_{0,0,2}X_1^2 + 18\psi_1X_1X_3}{I_{0,2,0}X_1^2X_3^2} - \frac{12\psi_3X_3^2 + 24\psi_5X_5X_3}{I_{0,2,0}X_1^2X_3^2}. \tag{15}$$

This is the general form of the soliton parameter dynamics for the pulse form given by (3). It remains to numerically solve these equations.

3. Numerical simulations

In this section, the dynamics of the six soliton parameters is followed for super-Gaussian and super-sech pulses, defined as: $f(\tau) = \exp(-\tau^{2m})$ and $f(\tau) = \operatorname{sech}^m(\tau)$. In these cases, different integrals (4) are given as (Table 1):

Table 1
Values of different integrals $I_{i,j,k}$ for super-Gaussian and super-sech pulses (4).

$I_{i,j,k}$	Super-Gaussian	Super-sech
$I_{2,2,0}$	$\frac{2^{\frac{2m-3}{2m}}}{2m}\Gamma\left(\frac{3}{2m}\right)$	$\frac{2^{2m-1}}{m^3}\,{}_4F_3(m, m, m, 2m; 1+m, 1+m; -1)$
$I_{0,0,2}$	$\frac{2m}{2^{\frac{2m-1}{2m}}}\Gamma\left(\frac{4m-1}{2m}\right)$	$-m^2\left[\frac{2mB(m,\frac{1}{2})}{2m+1} - \frac{2^{2m-1}(m+1)\Gamma^2(m)}{(2m+1)\Gamma(2m)} - \frac{2^{2+m}+1}{2+m}\,{}_2F_1(2+m, 2+2m; 3+m; -1)\right]$
$I_{0,2,0}$	$\frac{1}{m2^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)$	$B\left(m, \frac{1}{2}\right)$
$I_{0,3,0}$	$\frac{1}{m3^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)$	$B\left(\frac{3}{2}m, \frac{1}{2}\right)$
$I_{0,4,0}$	$\frac{4}{m2^{\frac{2m+1}{m}}}\Gamma\left(\frac{1}{m}\right)$	$B\left(2m, \frac{1}{2}\right)$

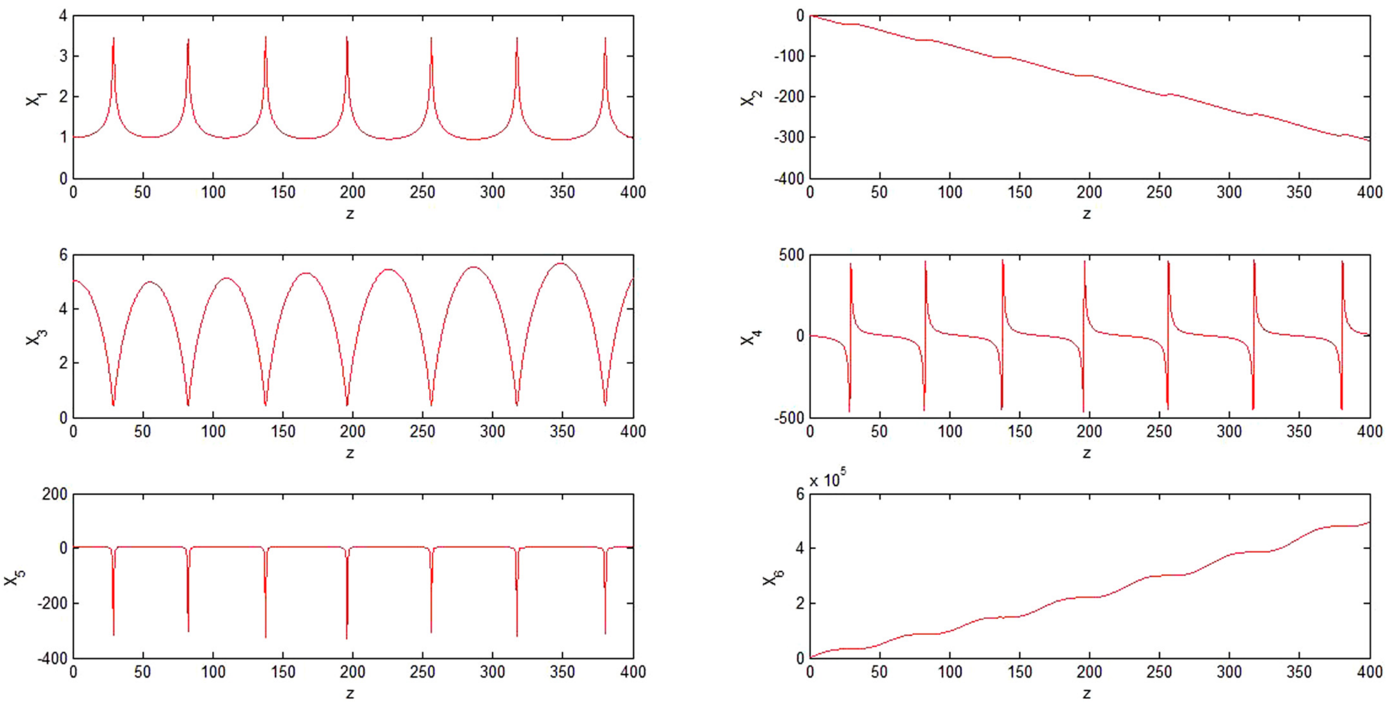

Fig. 8. Variation of super-sech pulse parameters, $m = 12$ (X_1-soliton amplitude, X_2-center position of the soliton, X_3-pulse width, X_4-soliton chirp, X_5-soliton frequency, X_6-soliton phase).

where the Gauss' generalized hyper-geometric function is defined as follows:

$$_pF_q\left(a_1, ..., a_p; b_1, ..., b_q; x\right) = \sum_{k=0}^{n} \frac{\prod_{j=1}^{p} (a_j)_k x^k}{\prod_{j=1}^{q} (b_j)_k k!} \tag{16}$$

and the beta function $B(l, m)$ is given by

$$B(l, m) = \int_0^1 x^l (1 - x)^m dx, \tag{17}$$

while the gamma function $\Gamma(x)$ is

$$\Gamma(x) = \int_0^{+\infty} e^{-u} u^{x-1} du. \tag{18}$$

These adiabatic evolution equations represent a versatile model of nonlinear dynamics. In fact, we have a system of ODEs and it requires a set of initial conditions. A numerical simulation allows us to present the propagation dynamics of the six parameters; we take $m = 2$, use the appropriate initial conditions and different values of ψ_j given in Table 2. Inspired by article [12], which studied quadratic-cubic nonlinearities in optical fibers by the method of collective variables, we choose specific values of different quantities in equation (1) for which the solution represents a localized optical soliton. We assume initial conditions $X_1(0) = 1$, $X_2(0) = 0$, $X_3(0) = 5$, $X_4(0) = 2$, $X_5(0) = 5$ and $X_6(0) = 0$. As mentioned, we utilize Runge–Kutta's fourth order method for numerical integration. The parameters in model (1) are chosen as: $a = 0.001$, $b_1 = 11$, $b_2 = 0.01$, $\alpha = 0.25$, $\lambda = 0.01$, $\theta = -0.1$.

In the end, our results are presented in a series of figures, Figs. 1–10, which depict various cases of beam characteristics. We evaluate the impact of the quadratic-cubic terms and present the dynamics of the six parameters for different values of m. Our results show that

Table 2
Values of different perturbations terms ψ_j for super-Gaussian and super-sech pulses (9).

ψ_j	Super–Gaussian	Super-sech
ψ_1	$-\frac{1}{2}\sqrt{2\pi}\alpha X_1 X_3 X_5 - \frac{1}{2}\sqrt{\pi}\lambda X_1^3 X_5$	$-\frac{8}{3}\alpha X_1 X_3 X_5 - \frac{64}{35}\lambda X_1^3 X_5$
ψ_2	$\frac{1}{4}\sqrt{\pi}\lambda X_1^4 X_3 X_4 + \frac{1}{4}\sqrt{\pi}\theta X_1^4 X_3 X_4$	$\frac{32}{35}\lambda X_1^4 X_3 X_4 + \frac{32}{35}\theta X_1^4 X_3 X_4$
ψ_3	$-\frac{1}{4}\sqrt{2\pi}\alpha X_1^2 X_5 - \frac{1}{8}\sqrt{\pi}\lambda X_1^4 X_5$	$-\frac{4}{3}\alpha X_1^2 X_5 - \frac{16}{35}\lambda X_1^4 X_5$
ψ_4	0	0
ψ_5	$-\frac{1}{2}\sqrt{2\pi}\alpha X_1^2 X_3 - \frac{3}{4}\sqrt{\pi}\lambda X_1^4 X_3 - \frac{1}{2}\sqrt{\pi}\theta X_1^4 X_3$	$-\frac{4}{3}\alpha X_1^2 X_3 - \frac{48}{35}\lambda X_1^4 X_3 - \frac{32}{35}\theta X_1^4 X_3$
ψ_6	0	0

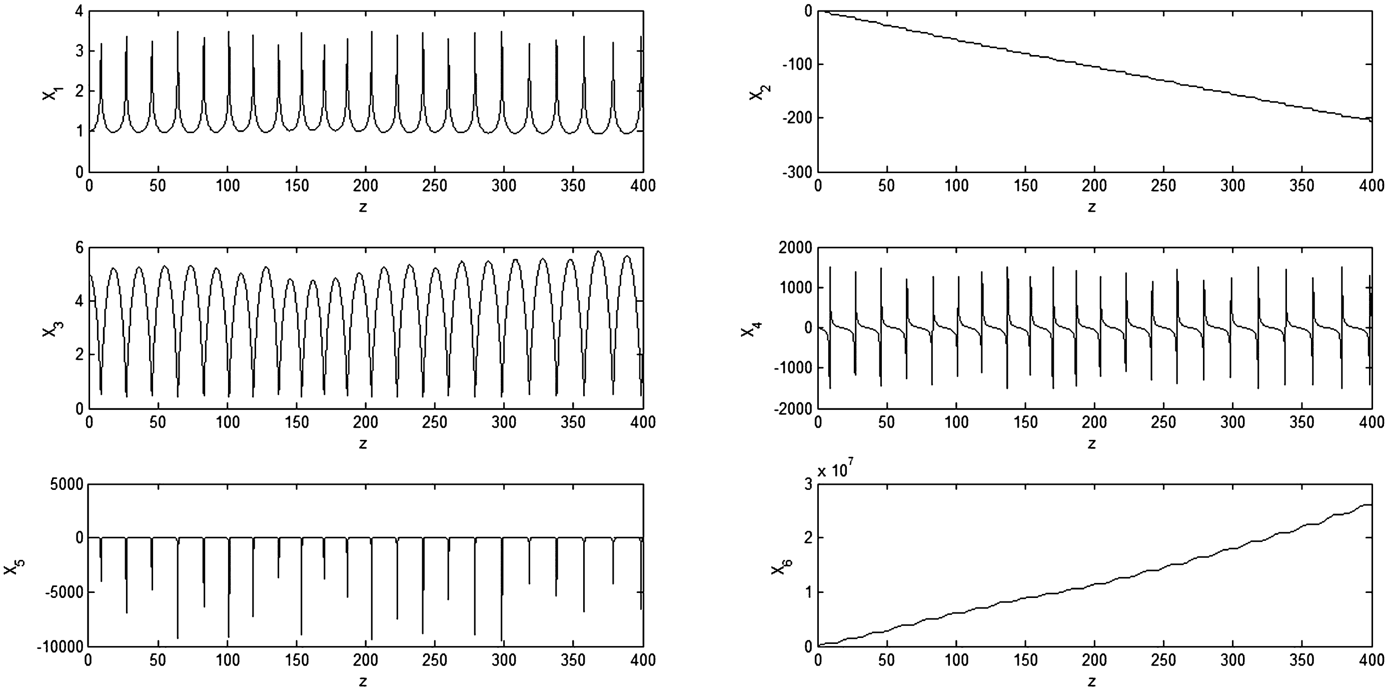

Fig. 9. Variation of super-Gaussian pulse parameters, $m = 1$ and b_1 changed to $b_1 = 110$ (X_1-soliton amplitude, X_2-center position of the soliton, X_3-pulse width, X_4-soliton chirp, X_5-soliton frequency, X_6-soliton phase).

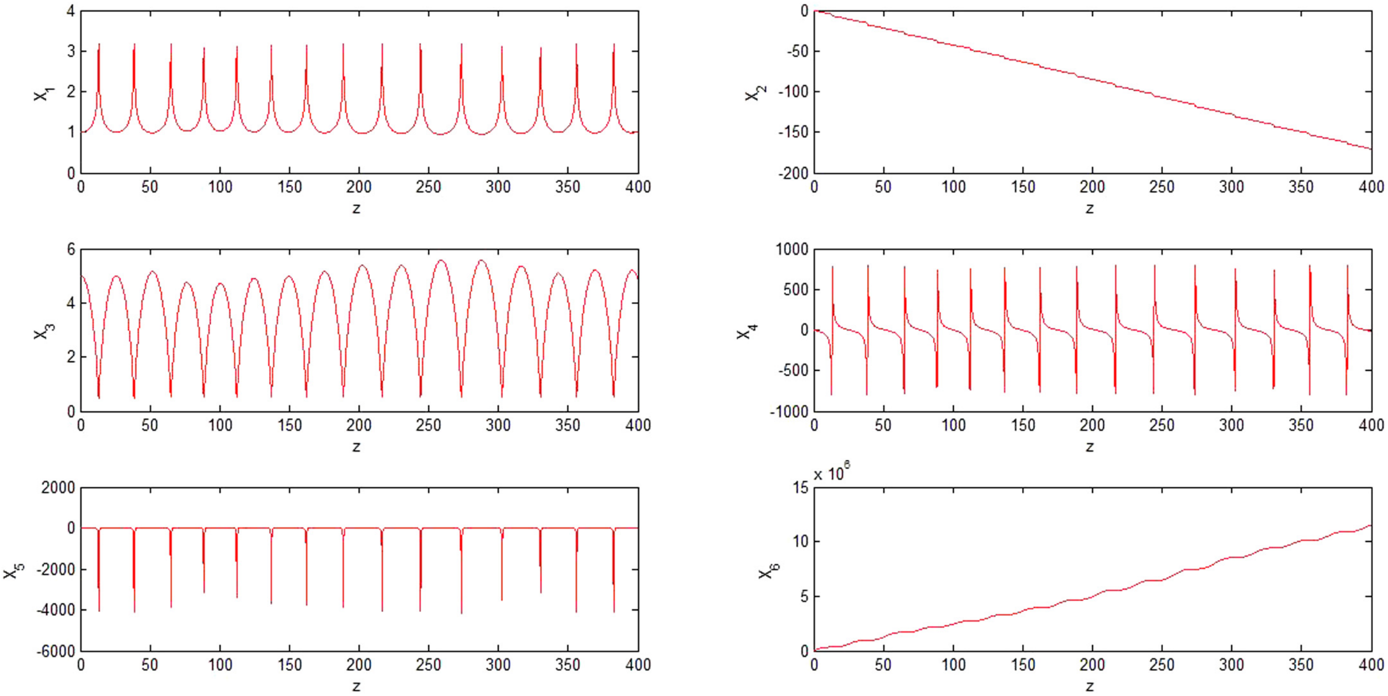

Fig. 10. Variation of super-sech pulse parameters, $m = 2$ and b_1 changed to $b_1 = 110$ (X_1-soliton amplitude, X_2-center position of the soliton, X_3-pulse width, X_4-soliton chirp, X_5-soliton frequency, X_6-soliton phase).

for $b_1 < -10$ the system does not oscillate but when b_1 increases, the oscillations start and their spatial frequency also increases. It is the same for b_2, where the oscillations are destroyed for $b_2 < 0$ but the spatial period decreases when this coefficient increases. Similar observations hold for the coefficients a and λ. On the other hand, the α coefficient exhibits little influence on the dynamics of the six parameters. It is noticed that the system does not oscillate for the values of λ and θ, as follows: $\lambda > 0.09$ or $\theta > 0.02$. The variational solutions corresponding to the initial super-Gaussian and super-sech conditions with $m = 4, m = 8, m = 12$ and for m greater than 12 showed a behavior similar to the solutions of initial figures $(1 - 3)$ with $m = 1$ and $m = 2$. The latter figures $(9 - 10)$ depict the change in the beam characteristics when the nonlinearity coefficient b_1 is substantially increased. The amplitude, the chirp, the width oscillate periodically within the prescribed regions.

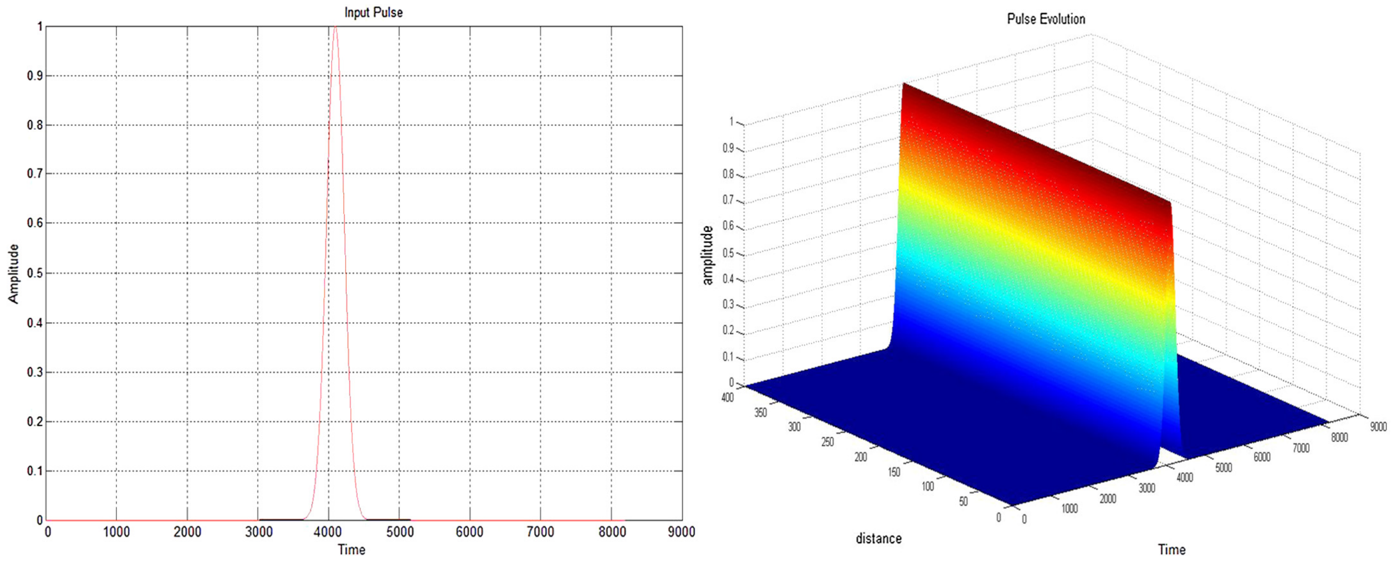

Fig. 11. Super-Gaussian Input Pulse and Pulse evolution for $m = 1$ by SSFM.

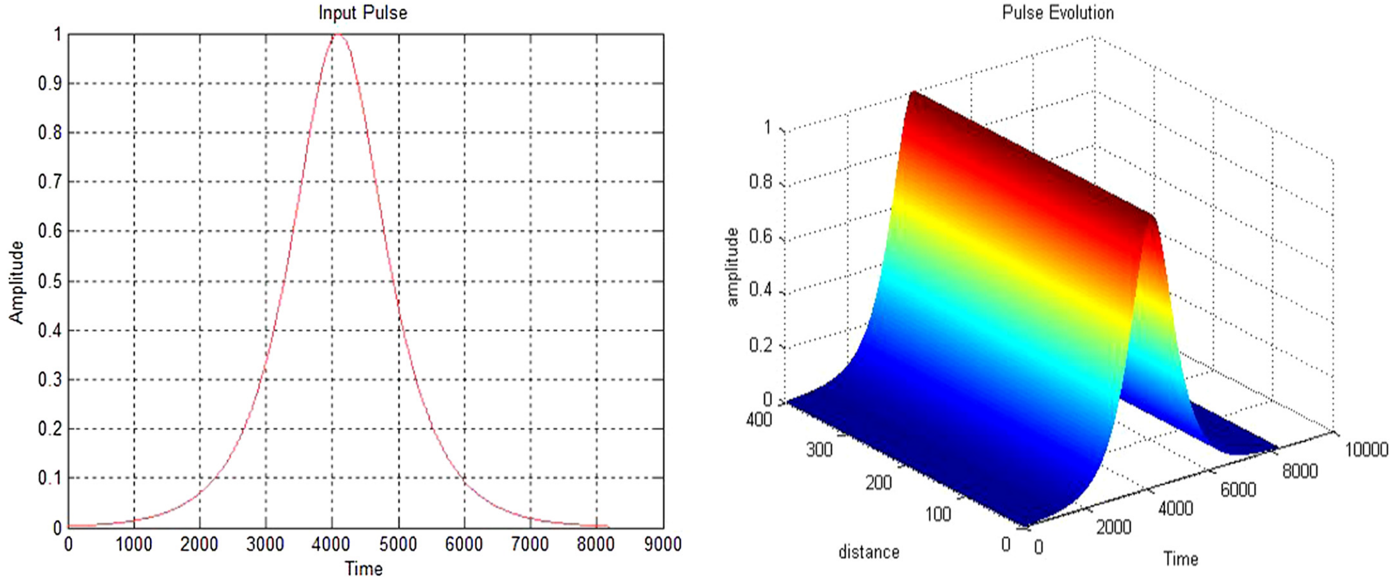

Fig. 12. Super-sech Input Pulse and Pulse evolution for $m = 1$ by SSFM.

We also execute a direct numerical simulation by solving Eq. (1) by Fourier method (SSFM). The results recovered are presented in Figs. 11–13. This method allowed us to present the pulse evolution in a 3D graph. These results confirm those obtained by the variational method for the same values of parameters b_1, b_2, a... in particular the amplitude which is a constant during the propagation. The impact of the coefficient a has been evaluated and presented in Fig. 13 where it can be seen that for given another values of the parameter a, the profile of the pulse is distorted. The super-Gaussian and super-sech pulses employed are a generalization of our previous work [12]. The results obtained are similar in both cases. We can therefore conclude that the stability of pulse propagation in these types of materials does not depend on the profile (super-sech or super-Gaussian) of the pulse.

4. Conclusions

Lagrangian Variational method is used to study QC nonlinearities. The super–Gaussian and super–sech pulses are chosen as the envelopes of the trial function. This method transforms the perturbed nonlinear QC Schrödinger equation into a system of coupled ordinary differential equations for the beam characteristics. The latter is handled by the fourth order Runge–Kutta method. Numerical simulations made it possible to present the dynamics of the six collective parameters (amplitude, center position, width, chirp, frequency and phase) for different values of parameter m and the nonlinear QC coefficients. These studies have shown that if parameter m increases, the frequency of the oscillation of the collective variables also increases. Similar observations were made for different values of the nonlinear QC coefficients. But for specific values of the nonlinear QC coefficients, the collectives variables may also not oscillate.

These soliton parameter dynamics are applicable to many aspects of the collective variables behavior, including but not limited to the collision induced timing and frequency jitter in solitons, four-wave mixing, ghost pulses, and the stochastic perturbation of solitons. Furthermore, they can be used to obtain analytical expression for soliton radiation.

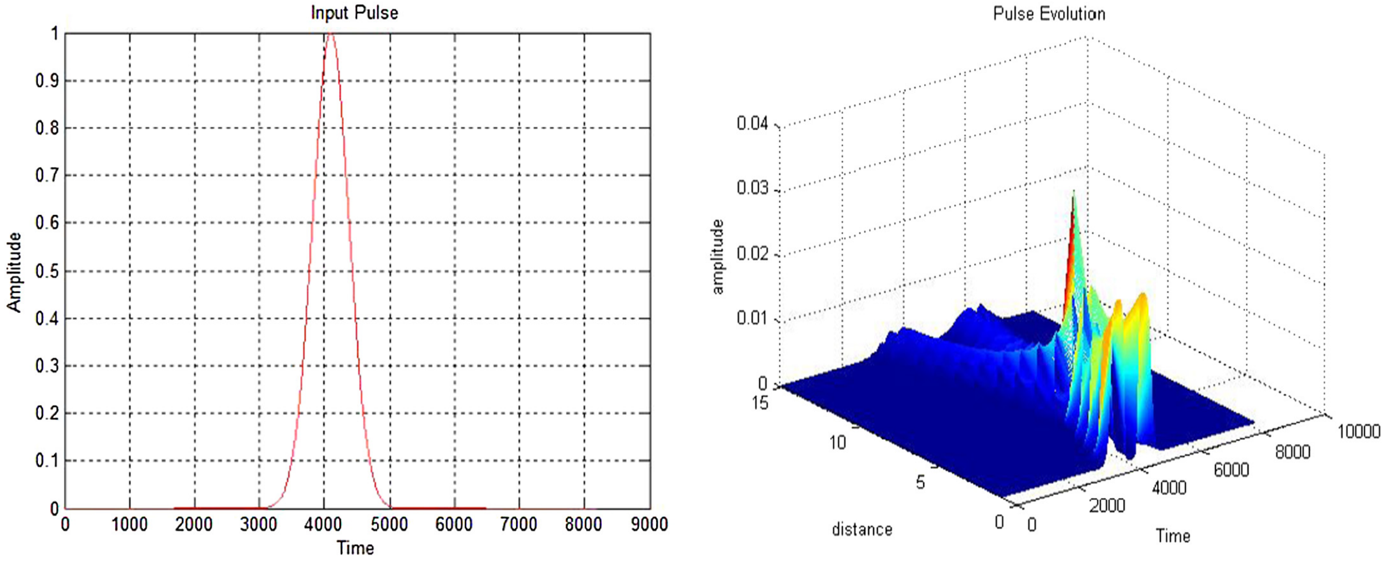

Fig. 13. Super-Gaussian Input Pulse and Pulse evolution for another value of $a = -20e - 27$ and $m = 2$ by SSFM.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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